lec. 6

Analog to Digital Converters

Analog input is quantized. Output is a digital code.

Performance metrics are similar to Track-Hold and DACs.

<table>
<thead>
<tr>
<th>Static:</th>
<th>Dynamic:</th>
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<tbody>
<tr>
<td>DNL</td>
<td>SNR</td>
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<tr>
<td>INL</td>
<td>SNDR</td>
</tr>
<tr>
<td>Gain error</td>
<td>SFDR</td>
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<tr>
<td>Offset error</td>
<td>Aperture jitter</td>
</tr>
<tr>
<td>Full-scale error</td>
<td>Settling time</td>
</tr>
<tr>
<td></td>
<td>Acquisition time</td>
</tr>
</tbody>
</table>
ADC DNL

differential nonlinearity (DNL) error

For an ADC, DNL error is defined as the difference between the ideal and the measured code transitions for successive codes. An ideal ADC would have finite digital codes exactly 1 LSB apart (DNL = 0). For a DAC, DNL error is the difference between the ideal and the measured output value between successive DAC codes. An ideal DAC would have analog output values exactly one code apart (DNL = 0). A DNL specification of greater than or equal to 1 LSB guarantees monotonicity (see monotonic).

ADC INL

Maxim AN-641
Full Scale Error

For an ideal ADC, the code edge triggers the transition to full-scale at 1.5LSB below the full-scale analog voltage. The full-scale error is the difference between this ideal code transition and the actual measured code transition. Full-scale error = offset error + gain error. See Figure 4.

Gain Error

Gain error is a figure of merit that indicates how well the slope of the actual transfer function matches the slope of the ideal transfer function. Gain error is usually expressed in LSB or a percent of full-scale range (%FSR). It can be calibrated out with hardware or in software.
For frequency domain applications, the accuracy of the ADC is limited by SNDR, just as was true for the DAC.

Define an effective # of bits (ENOB)

\[
ENOB = \frac{\text{SNDR}(or\text{SFD}) - 1.76}{6.02}
\]

So, taking the dynamic range that is given by harmonics, aliases, intermods and using that to judge the converter.

External noise.

Input wideband noise will be aliased into the Nyquist band. So, a low-pass filter must be used to attenuate external noise. Otherwise, SNR or SFDR will be compromised.
SNR = 6 \times 10\text{bits} + 1.76 = 62 \text{ dB} = \text{total noise power over Nyquist BW}

\[ ENOB = \frac{SINAD - 1.76}{6.02} = 9.5\text{bits} \]

F_{\text{sample}} = 62.35 \text{ MHz}

How can noise floor be – 95 dBc?

\[ \text{RBW} = \frac{\text{F}_{\text{sample}}}{2 \times 8192} \]

Noise floor reduced by 10 \log 8192 = 39 \text{ dB}
Oversampling

- Sampling frequency can be increased to spread quantization noise over wider Nyquist bandwidth.
- Then, noise power in a given signal bandwidth will be reduced.
Effective Resolution Bandwidth

- Increase input frequency at fixed $fs$
- ERBW = The frequency at which the SNR drops by 3 dB (1/2 LSB) compared with SNR at low input frequency.
Noise Figure

\[ F = \frac{(S/N)_{in}}{(S/N)_{out}} \]
\[ NF = 10 \log F \]

Nyquist ADCs are used in receivers, generally at IF, although for lower RF frequencies and lower SDR, in front ends. NF needed for system analysis.

We can calculate an effective noise figure for an ADC:

1. Assume white noise across Nyquist band.
   \[ f_{s}/2 \]

2. SNR is determined for continuous sinusoidal input signal 0.5 to 1 dB below full scale level.
   - Ideally \[ SNR = 6.02N + 1.76 \text{ dB} \]
   - But there are many factors that can degrade this —

3. Full Scale Power
   
   Peak voltage \( V_p \)
   
   Signal Power = \( S_{in} = \frac{V_p^2}{2R_{in}} \) (Watts)
   
   dBm = 10 \log(\text{Sin}) + 30 \text{dB}
4. Noise Power

\[
\text{SNR (dB)} \text{ in Nyquist BW}
\]

\[
\text{integrated noise power in BW} = \frac{f_s}{2}
\]

\[
\text{normalized noise power} = N
\]

\[
\text{Normalized to 1 Hz BW} \quad 10 \log \left(\frac{f_s}{2}\right)
\]

\[
\text{Thermal Noise} \quad 10 \log \left(\frac{1}{kT}\right) = -174 \text{ dBm/Hz}
\]

\[
S_{in} (\text{dBm}) - \text{SNR} - 10 \log \left(\frac{f_s}{2}\right) = \text{effective noise power normalized to 1 Hz BW.}
\]

So \(NF = \frac{N}{2} \text{, } N = (-174) \text{ dB}\)
Example:

1. Suppose $F_S = 2V$. $R_{in} = 50\Omega$

$$V_p = 1V \quad S_{in} = \frac{1}{100} = 0.01W = 10\text{mW} = 10\text{dBm}$$

Drop back 1dB $\rightarrow$ 9dBm

2. Suppose measured SNR for 12bit converter at 100Ms/s = 65dB
   (spurs are not included in NF calculation)

Nyquist BW noise power $= 9 - 65 = -56\text{dBm}$

$$10 \log \left( \frac{F_S^2}{2} \right) = 10 \log (5 \times 10^3) = 77\text{dB}$$

3. Normalized Noise Power $= -56\text{dBm} - 77\text{dB}$
   $= -133\text{dBm}$

4. $NF = -133 + 174 = 41\text{dB}$
   
   (better not put this one out the front end!)
What happens if you reduce the sampling frequency?

Same noise is spread over a narrower bandwidth.

10 dB/decade increase in NF.

So you would expect to see SNR improve with $f_s$ up to the point where jitter or TH glitches start to dominate.

\[ \text{SNR} \]

\[ -10 \text{dB/decade} \]

\[ \log f \]

\[ f_{s/2} \]
Flash ADC, n-bits

Full parallel architecture for speed.

2^n comparators + latches
2^n resistors

Comparator outputs are thermometer coded.
act as parallel sample-hold

output = 1 if Vin > VREF
= 0 if Vin < VREF
metastable if Vin ≈ VREF

may not require THA at front-end.
large number of components generally limits flash converters to 8 bits or less.

Errors:

1. Reference static and dynamic errors.
   A. R resistor mismatch.
   
   Same considerations as in resistor chain DAC.
   
   B. Bias currents.
   
   Bipolar comparator

\[ \text{Ref. ladder} \]

Signal feedthru

base currents create INL - having each tap voltage is decreased.

C. Signal feedthru is worse at higher freq.
   
   Disturbs reference voltages dynamically.
Fig. 7.21

Fig. 7.23  Generation of kickback noise in a bipolar comparator.

Fig. 6.13  Thermometer-binary decoder with a sparkle error.

should be 1000 → instead we get 1111
half of full scale error
D. Nonlinear input capacitance.

Lots of comparators: input capacitance can be large and nonlinear.

Voltage dependent phase shift.

Causes harmonic distortion.

\[ \begin{align*}
\text{Rs} & \quad \text{Vout} \\
\text{Vin} & \quad \text{Cin} (\text{Vout})
\end{align*} \]

E. Kickback. When comparators are stressed, noise is generated at their inputs.

(Fig. 7.21, 7.23)
F. Comparator aperture uncertainty.

- Comparators may not be identical or
- Clock delay causes a lower comparator to latch earlier than the one above.

![Comparator Circuit Diagram]

"Sparkle" when \( V_{\text{in}} \) has high slew rate,

![Slew Rate Graph]

can also be caused by DC offset in comparator.

Figs. 6.12, 6.13 Razavi,
can cause huge errors in \( \text{DMS} \to \text{binary} \) conversion. Degrades SNR, creates distortion.
Lec. 7

Last time:

ADC static performance
dynamic performance

\[ \text{ENOB} = \frac{SFDR - 1.76}{6.02} \]

Noise figure

Flash ADC architecture
errors:

- mismatch
- loading
- delay on RC input network
- bubbles or sparkles in therm. code

Today:

Comparator metastability

2-step converters
interpolation
Since linearity isn't needed, use positive feedback in the latch to effectively increase the gain.

This also is used to define the sampling instant (like the track-hold)

\[ V_1 \rightarrow V_T \rightarrow \text{Latch} \rightarrow V_{\text{out}} \]

**Track:** \( CK = 1 \)  \[ V_T \text{ tracks input; latch is disabled} \]

**Latch:** \( CK = 0 \)  \[ \text{amp. is disabled; latch holds present value of } V_T. \text{ Senses polarity.} \]

This works well when \( V_1 - V_2 \) is not small. When small we can have metastable case when latch can take a long time (or never) to respond, dynamic error.
Ideal comparator:

\[ V_{\text{out}} = \begin{cases} 
0 & \text{if } V_1 - V_2 \leq 0 \\
V_{\text{sat}} & \text{if } V_1 - V_2 > 0 
\end{cases} \]

can be approximated by a high gain amplifier.

Output is valid when \( |V_1 - V_2| > \frac{V_{\text{th}}}{A_v} \)

so, resolution depends on gain.
Likewise,
\[ N_x + T_0 \frac{dN_x}{dt} = -A_0 N_y \]

Subtract
\[ T_0 \frac{d(N_x - V_y)}{dt} = -(1-A_0)(N_x - V_y) \]

Initial condition \( V_{xy0} = (N_x - V_y) \bigg|_{t=0} \)

\[ V_x - V_y = V_{xy0} \exp \left[ (A_0 - 1) \frac{t}{T_0} \right] \]

growing with time since \( A_0 \gg 1 \)

**BUT**!

with time constant \( T_0 / (A_0 - 1) \)

and if \( V_{xy0} \) is very small, it could take a long time to reach valid logic levels.
Simple latch model.

\[ \tau_0 = \tau C \]

Represent as single dominant pole for determining transient response.

\[ \frac{N_x - N_2}{R} - C \frac{dN_2}{dt} = 0 \]

\[ -N_2 - \tau_0 \frac{dN_2}{dt} = -N_x \]

\[ N_2 = -\frac{N_y}{A_0} \]

\[ \frac{N_y}{A_0} + \frac{\tau_0}{A_0} \frac{dN_y}{dt} = -N_x \]

\[ N_y + \frac{\tau_0}{A_0} \frac{dN_y}{dt} = -A_0 N_x \]
Starting at metastable point $V_{MS}$, we must reach $V_{IH}$ or $V_{IL}$ before logic is fully stable.

How long will this take?

$$T_i = \frac{T_o}{A_0 - 1} \ln\left(\frac{V_{IH}}{V_{XYO}}\right)$$

If $V_{XYO}$ is very small, this can take a long time. Then, a sparkle can be produced because a valid logic state has not been obtained yet.

What is the probability?

If $T_c = \text{regeneration time} = \frac{1}{2} T$,

$$P\left(T_i > T_c\right) = \exp\left(-\frac{(A_0 - 1)T_c}{T_o}\right)$$

Increase $A_0$, decrease $T_o$. 
The effective gain in the latch stage can be increased by cascading.

For single latch stage,

\[ V_0 \approx A_{\text{pre}} V_{\text{in}} \exp\left(\frac{t}{C}\right) \]

where \( V_0, V_{\text{in}} \) are usually differential and \( C = \frac{C_0}{(A_0 - 1)} \)

\( t \approx \frac{1}{2} \quad \left( \frac{1}{2} \text{ period of clock} \right) \)

By adding a second latch in cascade, we increase the regeneration time by another half clock cycle.

Now,

\[ V_0 \approx A_{\text{pre}} V_{\text{in}} \exp\left(\frac{t_1}{C}\right) \exp\left(\frac{t_2}{C}\right) \]

\[ = A_{\text{pre}} V_{\text{in}} \exp\left(\frac{t_1 + t_2}{C}\right) \]

A third latch stage can also be used to further increase gain at the cost of latency.

8-bit Flash ADC


Pipeline detail

Reduced error rate due to cascaded latches


ENOB vs. input amplitude

Fig. 10. Effective number of bits versus full-scale range for 1-, 30-, and 100-MHz analog input frequencies, all at 250 mV/p. Input amplitude was adjusted for each range.

Flash ADCs have a practical limit of 8 bits or so. How can we get higher resolution?

Tradeoff speed or latency for complexity reduction and accuracy.

Two-step ADCs

t₁: coarse analog estimate
  + MSB quantization
  take difference

t₂: fine LSB A/D using flash

TH is needed because of time delay between t₁ and DAC settling. Otherwise we have an error ΔV. This would greatly limit maximum input frequency to well below Nyquist.
Requirements.

1. Coarse DAC must have high precision
   $\pm \frac{1}{2}$ LSB
   otherwise fine ADC receives incorrect residue.

2. Much slower than flash ADC
   SH/TH stage much slower than comparator.
   Comparator just requires 0 or 1 output whereas the TH must be precise and fully settled.
   
   Must wait also for DAC, subtractor, and fine AD to completely settle.

3. Fine ADC has small full scale value, less dynamic range
   you can amplify output of subtractor, but that adds more requirements for accuracy and distortion of the amplifier.

\[ \text{Diagram: } \]

- [Diagram of circuit with subtractor, amplifier, and fine ADC]
Nonidealities.

Coarse stage static error carries through since the coarse estimate and the signal are subtracted (analog). So, DNL, INL, offset and gain errors are all deadly.

This difference is called a residue. A plot of this residue over a swept input can reveal the error type:

\[(R_{\text{err}})\]

In order to obtain a better view of these effects, the concept of the quantization error plot (Section 6.1) can be generalized to a residue plot wherein the difference between the actual and ideal characteristics is depicted as a function of the input (Figure 6.27). In a residue plot, gain error appears as peaks that follow a nonhorizontal straight line, DNL as shifted transition points, INL as peaks that do not follow a straight line, and offset as a vertical shift [Figure 6.27(b)].

![Residue plot](image)

\[V_{\text{in}}\]

(a)

![Residue plot](image)

\[V_{\text{in}}\]

(b)

**Fig. 6.27** Residue plot derived from characteristic of Figure 6.26. (a) Ideal; (b) including errors.
If these errors exceed 1 LSB, you can actually have missing codes because the subtractor output might not reach the full m-bits of the coarse ADC before it switches to the next coarse code.

See book for example (p 123)

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Fig. 6.28  (a) Subtractor output in a two-step ADC; (b) missing code; (c) nonmonotonicity.
2 stage 10 bit ADC

94 comparators instead of 1024

Figure 1: Block diagram of a 10-b, 1GS/s ADC.


2 stage ADC dynamic performance

Figure 5: Measured ADC performance at 500 MS/s with 49 MHz input.

Figure 6: Measured ADC performance at 500 MS/s with 200 MHz input.

Subranging ADCs.
+ No subtractor.

Coarse stage subdivides reference
Fine stage compares against subdivided reference.

Fig. 6.31 Section of a 10-bit subranging ADC architecture.

Problems:
1. R ladders are slow to settle
2. Comparators must operate over the full input common mode range.
Interpolation.

Make use of linearity of comparator preamp to obtain extra resolution while retaining one-stage ADC topology.

Fig. 6.33 Interpolation in a flash ADC.

How it works: plot amplifier outputs

Note that the relative values of \( v_{X2} \) and \( v_{Y1} \) exactly represent the difference between \( V_{IN} \) and \( V_m = \frac{V_{R1} + V_{R2}}{2} \). So, we interpolate between two ref. levels. Latch senses polarity and gives extra bit.
You can increase the degree of interpolation by using a resistor chain to further increase the bits between two taps of a flash converter.

Fig. 6.36 Higher order interpolation. (a) Implementation; (b) input/output characteristics.
Folding ADC Architecture

2-step ADCs split the data conversion into two or more time phases.

Delays of SHA, Coarse ADC, DAC and subtractor must all settle before residue is valid. Residue

Subranging eliminates the DAC and subtractor but still is a two-step process.

C-ADC activates switches to select subrange for FA-ADC.
Folding is an approach which generates the residue in a one-step process.

\[ \text{Vin} \rightarrow \text{Flash CADC} \rightarrow \text{MSBs} \]
\[ \text{Analog Preprocess} \rightarrow \text{Flash FADC} \rightarrow \text{LSBs} \]
\[ \text{parallel}\]

Fig. 6.40 (a) Input/output characteristics of two amplifiers; (b) sum of characteristics in (a); (c) residue in a two-step ADC.

\[ R_{C1} = R_{C2} = R, \ I_1 = \cdots = I_5 = I \]

Fig. 6.41 Folding circuit.