

## Sources of errors in digital communication

Intersymbol Interference (ISI)

Additive Noise

Jitter

Data bits get corrupted and fail to produce a clean 0 or 1. This produces bit errors. A good digital transmission link should have a BER  $< 10^{-12}$ .

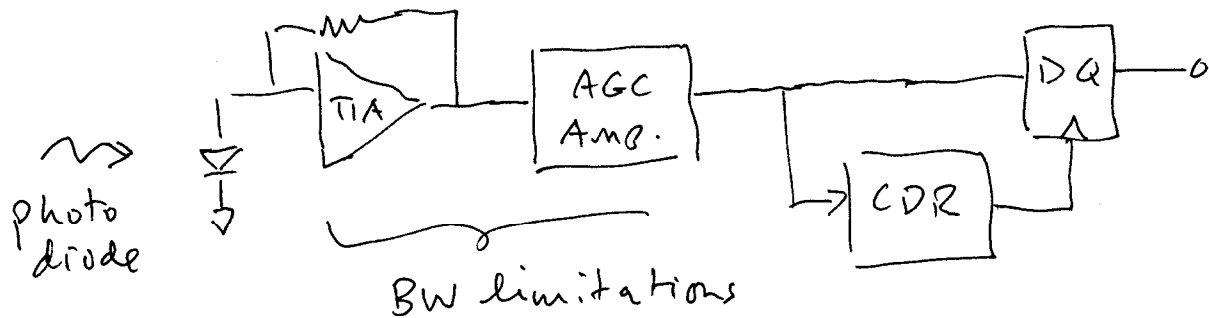
ISI. Data errors that are pattern dependent can be produced by bandwidth limitations. Periodic data does not experience this. AKA: DDJ, PDJ

Low Pass Filter. Reduced BW. This produces exponential rise/fall times. If these affect the next bit period, amplitude and threshold ~~errors~~ distortion will potentially produce errors.

$$V(t) = V_0 [1 - \exp(-t/\tau)]$$

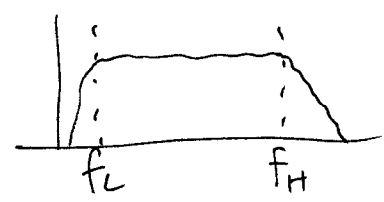
~~$V(t) = V_0 [1 - \exp(-t/\tau)]$~~

Ref. B. Razavi, Design of Integrated Circuits for Optical Communications, McGraw-Hill, 2003.



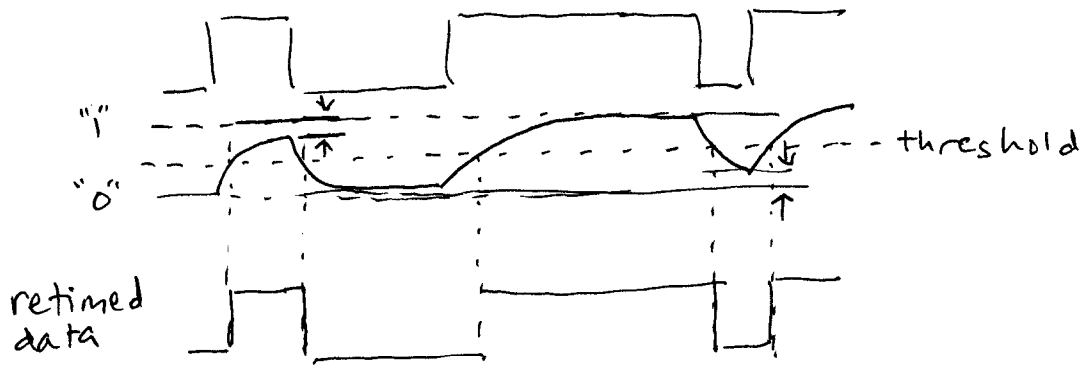
$$f_L, f_H$$

$$BW = f_H - f_L \cong f_H$$

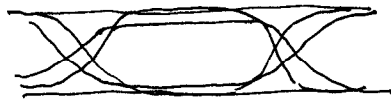


$$f_L = \frac{1}{2\pi R_L C_L} \quad \text{High Pass}$$

$$f_H = \frac{1}{2\pi R_H C_H} \quad \text{Low Pass}$$



pulse width distortion, weak 0 or 1



To predict amplitude reduction for given BW:

$$\ln\left(\frac{V(t)}{V_0} - 1\right) = -\frac{t}{\tau}$$

$$f_{3dB} = \frac{1}{2\pi\tau}$$

EX:

$$T_b = 100ps$$

$$f_{3dB} = 3GHz$$

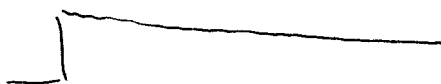
$$\tau = 53ps$$

$$\frac{V(t)}{V_0} = 1 - \exp\left(-\frac{T_b}{\tau}\right) = 1 - 0.15 = 0.85$$

So 15% reduction in 1 bit period

2.3% in 2 bit periods

AC coupling to eliminate DC offsets  
also can produce bit errors through droop.



$$V(t) = V_0 [\exp(-t/\tau)]$$

For long sequences of 0 or 1's, a very  
low HPF cutoff may be required.

EX:  $f_L = 1 \text{ MHz}$ .  $\tau = 159 \text{ ns}$

How many bit periods ( $T_b = 100 \text{ ps}$ ) to get 1%  
error in  $V_0$ ?

$$\frac{V}{V_0} = 0.99 = \exp(-t/159 \text{ ns})$$

$$- 159 \text{ ns} \cdot \ln(0.99) = t = \frac{1.6 \text{ ns}}{1.6 \text{ ns}} \quad \text{or } \frac{1600 \text{ bits}}{1600 \text{ bits}}$$

~~highly unlikely~~

16 bits

## Pulse Width Distortion

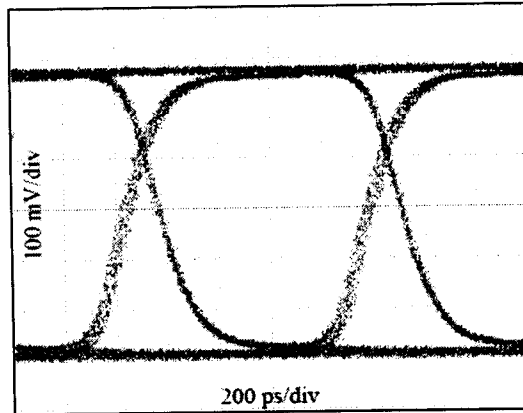


Figure 4. Eye diagram showing PWD

## Intersymbol Interference (ISI)

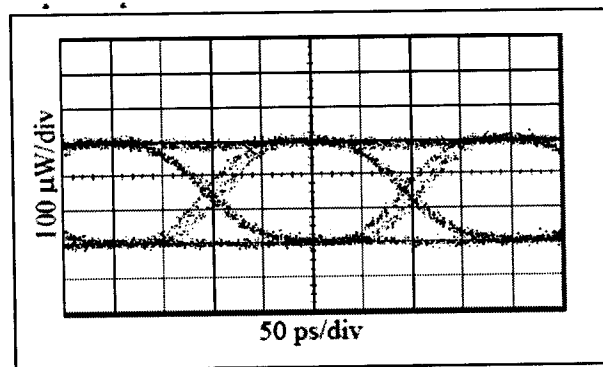
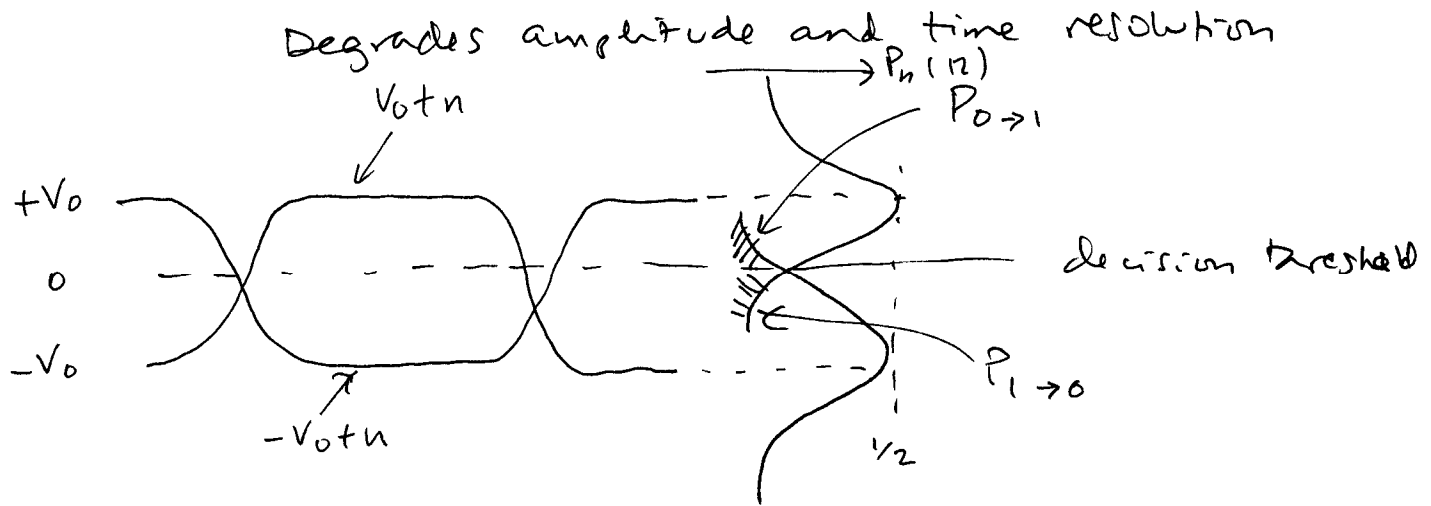


Figure 5. Eye diagram showing DDJ/ISI/PDJ

Jitter in Digital Communications Systems, Part 1,  
Maxim App. Note 1/FAN 4.0.3, Sept. 2001.

## Additive Noise



Assume Gaussian noise PDF

$$P_n = \frac{1}{\sigma_n \sqrt{2\pi}} \exp\left(-\frac{n^2}{2\sigma_n^2}\right)$$

Assume 0 and 1 have equal probability.

$$P_{0 \rightarrow 1} = \frac{1}{2} \int_0^{\infty} \frac{1}{\sigma_n \sqrt{2\pi}} \exp\left(-\frac{(u+V_0)^2}{2\sigma_n^2}\right) du = P_{1 \rightarrow 0}$$

lets substitute  $z = (u+V_0)/\sigma_n$

$$P_{0 \rightarrow 1} = \frac{1}{2} \int_{V_0/\sigma_n}^{\infty} \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{z^2}{2}\right) dz \quad \left[ \begin{array}{l} \text{error} \\ \text{function} \\ \text{erfc}\left(\frac{V_0}{\sigma_n}\right) \end{array} \right]$$
$$= \frac{1}{2} Q\left(\frac{V_0}{\sigma_n}\right)$$

$$P_{\text{TOTAL}} = Q\left(\frac{V_0}{\sigma_n}\right)$$

$\frac{V_0}{\sigma_n}$  is essentially the SNR

$\sigma_n$  requires knowledge of the BW. Noise must be integrated over signal BW.

This probability gives us the BER.

for  $x > 3$ ,  $Q(x) \approx \frac{1}{x\sqrt{2\pi}} \exp\left(\frac{-x^2}{2}\right)$

So,  $\frac{V_0}{\sigma_n} = 5$  gives us  $Q(5) \approx 3 \times 10^{-7}$

3 bits per  $10^7$  in error

$$Q(6) \approx 10^{-9}$$

$$Q(7) \approx 10^{-12}$$

since  $\sigma_n \propto \sqrt{\text{BW}}$ , we must avoid excessive BW.

on the other hand, ISI requires sufficient BW

$$\text{BW} \approx 0.7 \times \text{Data Rate}$$

is good compromise for NRZ data.

## Jitter.

Many definitions, but all refer to a time difference between the ideal time for an event to occur and the actual time it occurred.

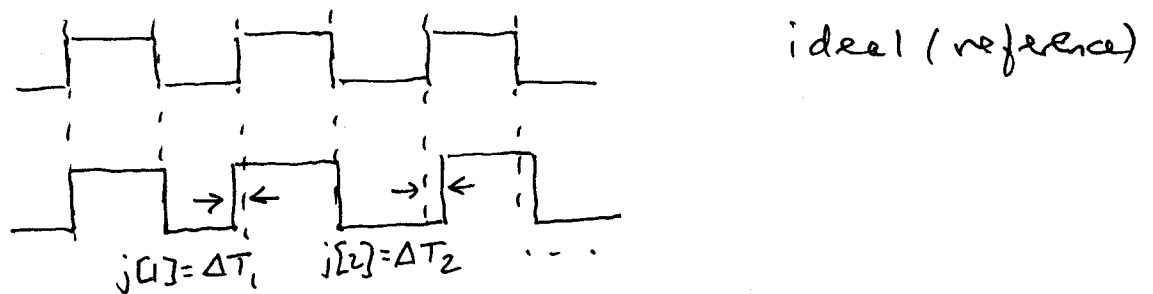
The event could be

- sampling instant of data (NRZ)
- zero crossing of differential signal
- rising or falling clock edge

## Instantaneous or Absolute Jitter.

Deviation of event from ideal

$$j[n] = t_{\text{ideal}} - t_{\text{Actual}}$$





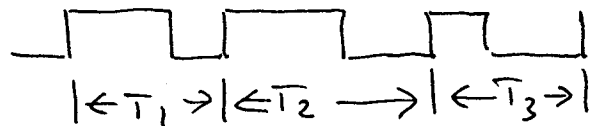
Average jitter. (rms method)

$$\mu = \frac{1}{N} \sum_{n=1}^N j[n]$$

$$\sigma = \sqrt{\frac{1}{N-1} \sum_{n=1}^N (j[n] - \mu)^2}$$

peak-to-peak jitter =  $\max\{j[n]\} - \min\{j[n]\}$

Cycle-to-cycle jitter



$$\Delta T_{cc} \cong \lim_{N \rightarrow \infty} \frac{1}{N} \sqrt{(T_2 - T_1)^2 + (T_3 - T_2)^2 + \dots}$$

no reference (ideal) is required.

Two types of Jitter:

Random (RJ)

Gaussian PDF describes jitter  
unbounded

Deterministic (DJ)

caused by periodic interferences

Power / Ground noise

<sup>cross-</sup>oscillator coupling

ISI, duty cycle distortion

bounded

RJ comes from phase noise of oscillators  
and the effects of additive noise  
on the data threshold.

## Deterministic Jitter (DJ)

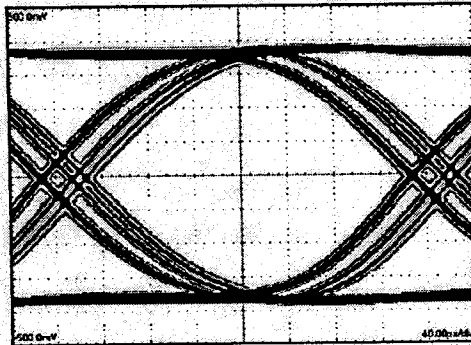


Figure 3. Eye diagram showing DJ

## Random Jitter (RJ)

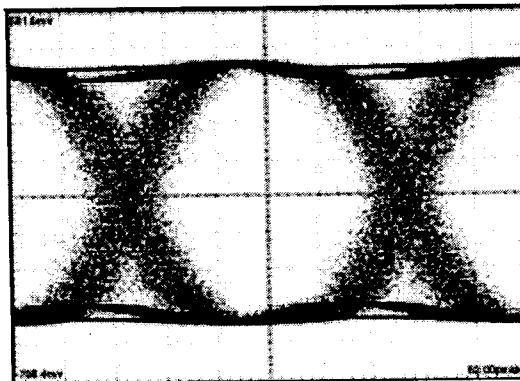


Figure 2. Eye diagram showing RJ

Jitter in Digital Communications Systems, Part 1,  
Maxim App. Note HFAN 4.0.3, Sept. 2001.

## DJ

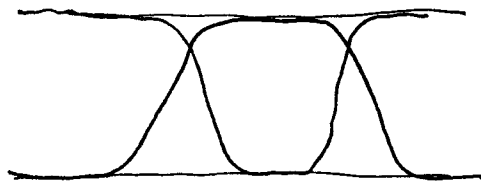
### Duty Cycle or Pulse Width Distortion

unequal width of 0 and 1

unequal rise/fall times can cause this.

~~for~~ voltage offsets on differential inputs

crossover is shifted up or down



0 is shorter than 1

### Data Dependent Jitter (DDJ)

same as ISI

or also known as Pattern Dependent J.  
(PDJ)

### Sinusoidal Jitter

$$j[n] = A \sin \left[ 2\pi f \frac{n}{r} + \phi \right]$$

$r$  = data rate       $f$  = jitter freq.

$n$  = cycle #

This can be caused by oscillator coupling through power/gnd or substrate.

RJ.

peak-to-peak measurements are not meaningful because in theory Gaussian tail is infinite.

Limits based on the required BER can be used to convert from rms to pk-pk jitter.

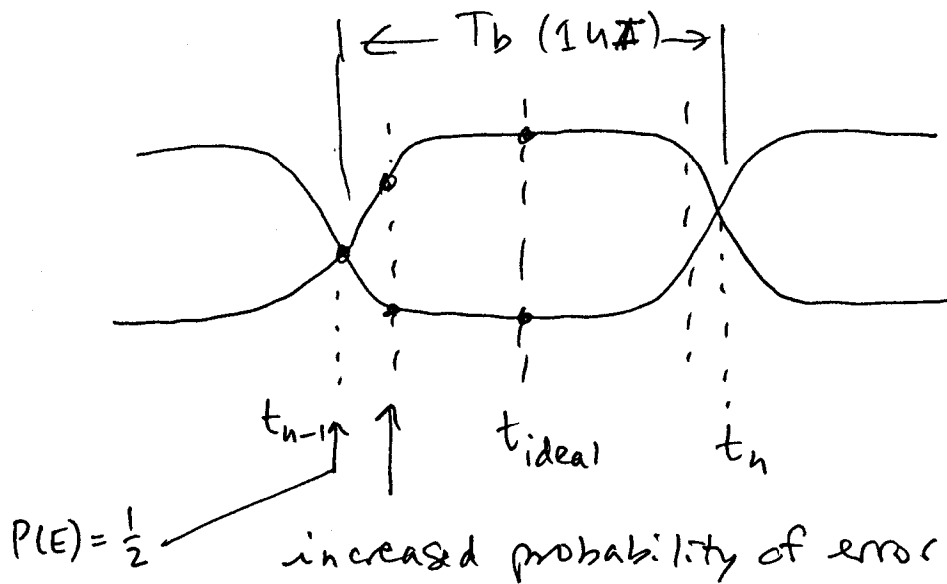
Since DJ is bounded, it typically is expressed as pk-pk.

$$\text{Total RMS jitter} = \sqrt{\sigma_1^2 + \sigma_2^2 + \dots}$$

where each  $\sigma_i$  comes from an individual system component.

How will jitter affect BER?

→ Timing of sampling instant is uncertain.



Now  $P(E)$  must take into account the probability of bit error at each sampling instant:

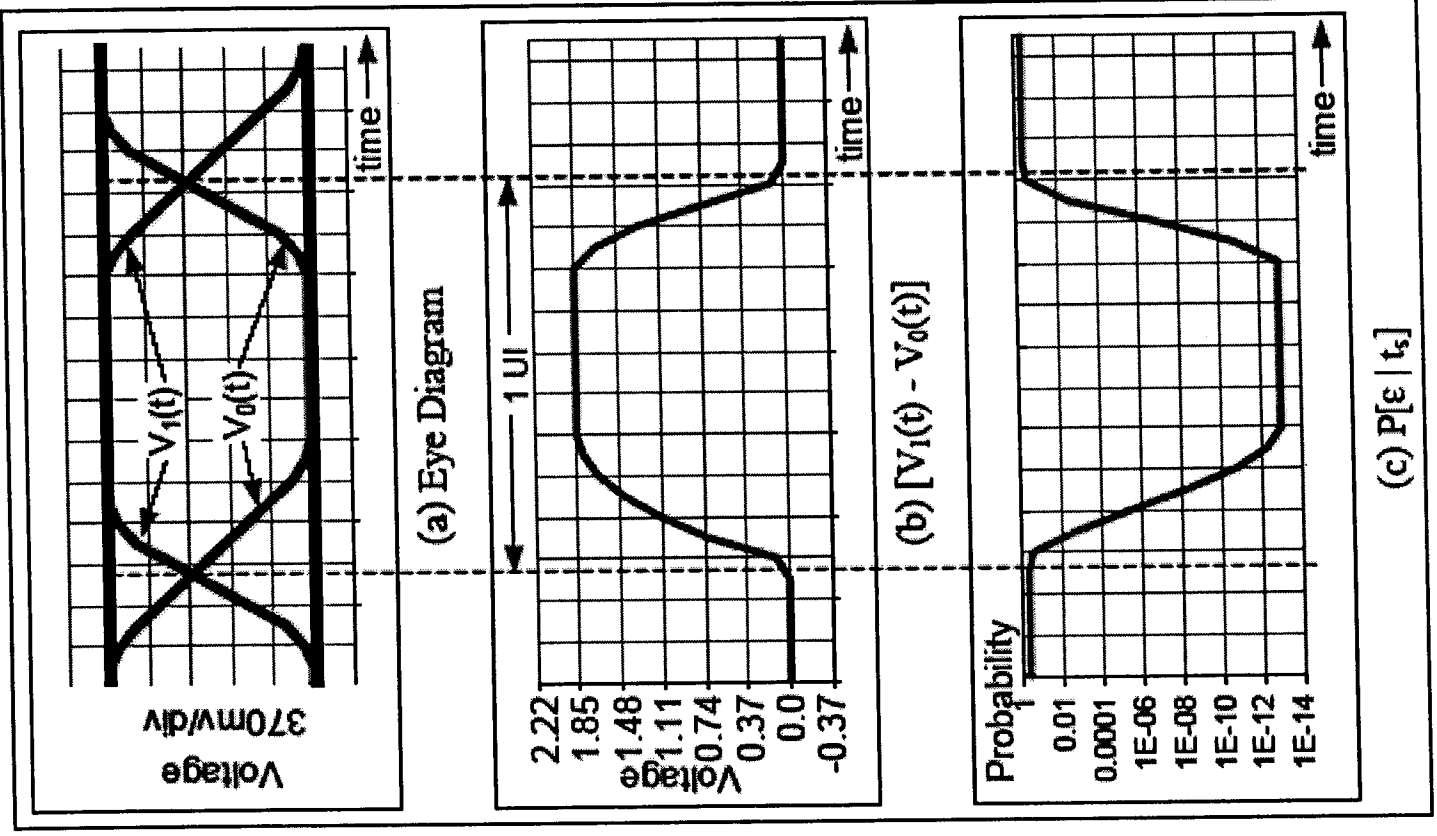
$$P(E | t_s)$$

as well as the probability that sampling instant  $t_s$  actually occurs  $P[t_s]$ .

consider jitter to be on clock edge, not data edge.

Assume noise  $\delta n$  on both 0 and 1, Gaussian RJ

"Bath tub" Plot



averages:  
 $\bar{V}_1 - \bar{V}_0$

probability of bit error at each  $t_s$

Timing probability  
of bit clock  
relative to data

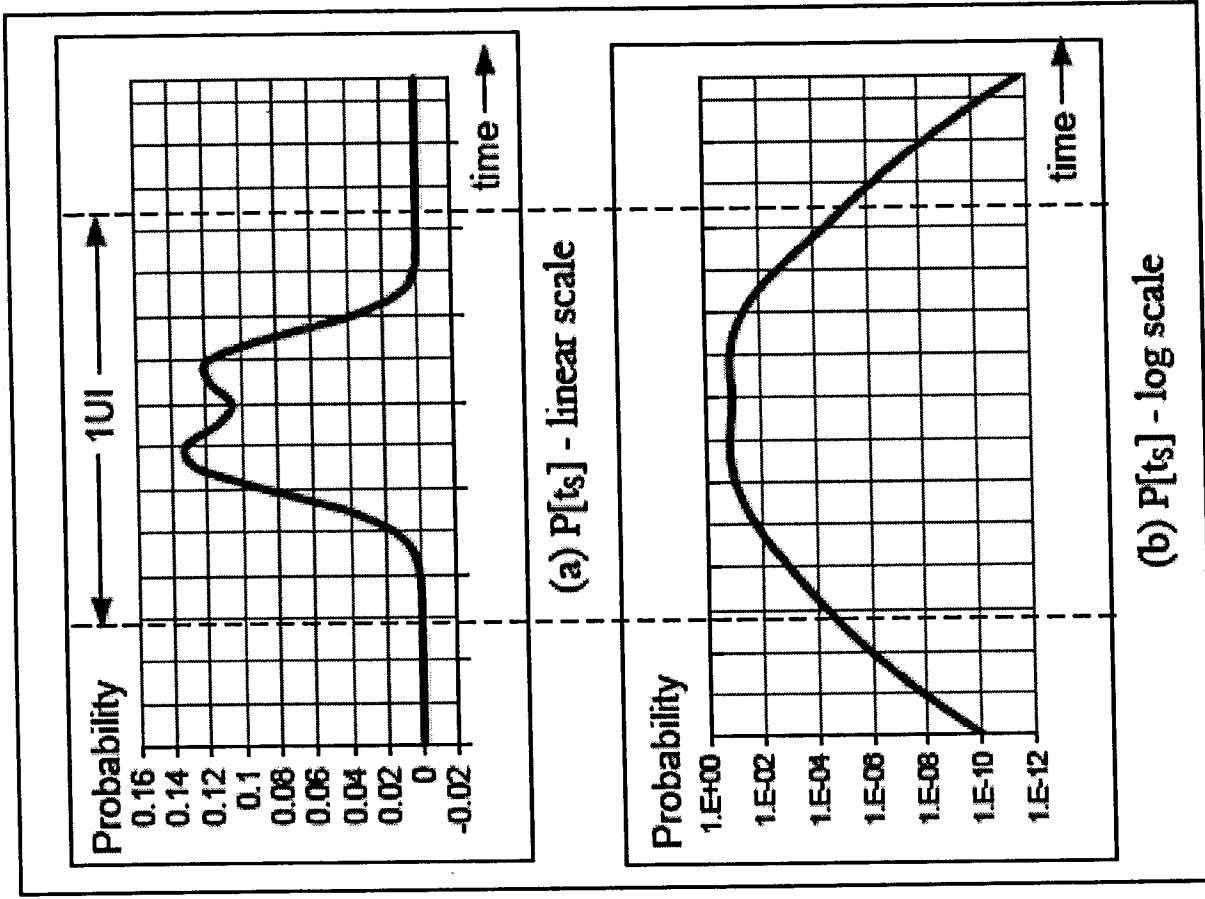


Figure 7. Sampling instant probability density function (PDF)

Maxim, sp. cit



w

Maxim, op.cit.

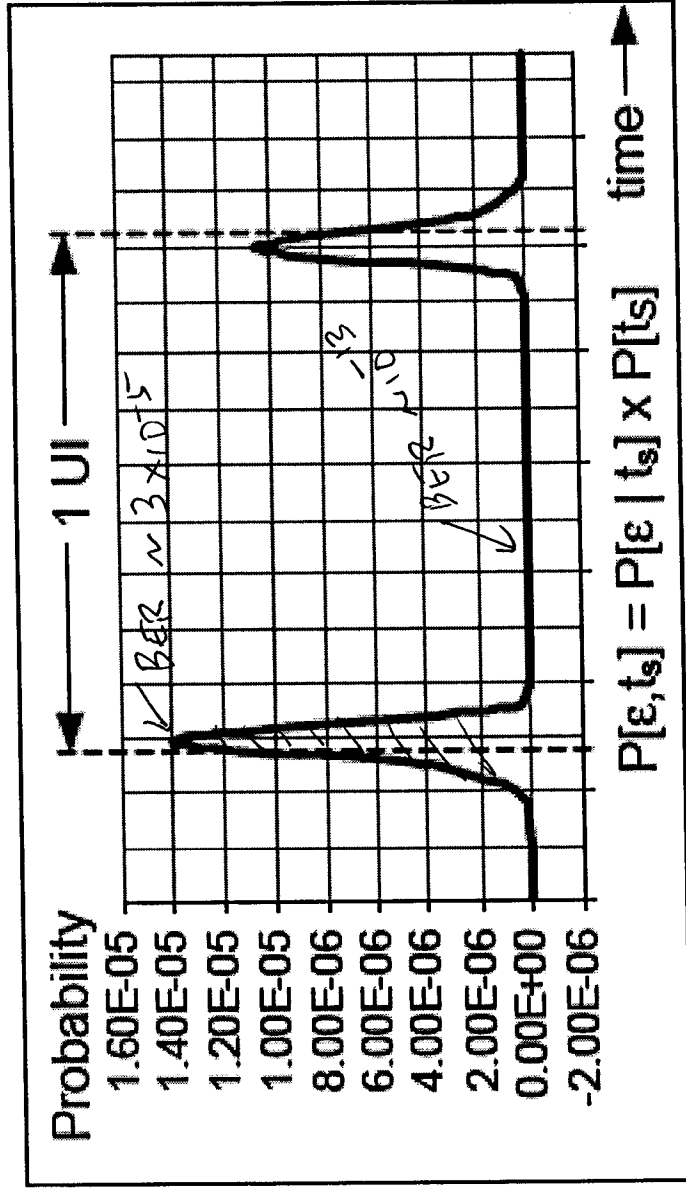


Figure 8. Probability of bit error over the range of possible sampling instants

Multiply bathtub plot with  $P[t_s]$ .

$$BER = \int_{-\infty}^{\infty} P[e, t_s] dt$$

## Jitter Peaking

The TYPE 2 loop filter with an ideal integrator gives

- zero steady state phase error for a  $\Delta\omega$
- loop BW independent of phase error
- damping improves as  $\omega_2$  is reduced for fixed  $\omega_n$
- crossover frequency is increased

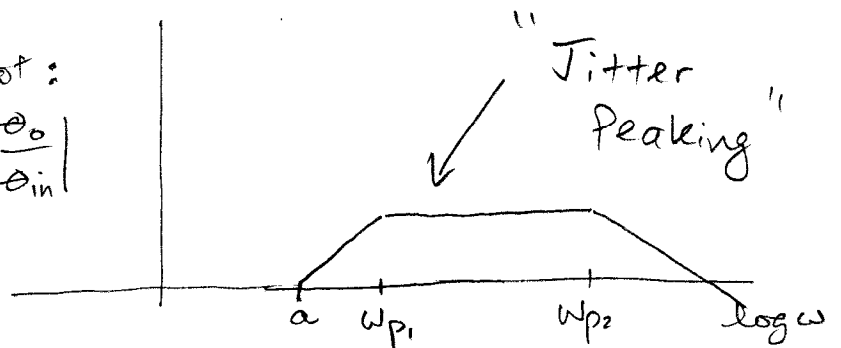
$$T(s) = \frac{K(s+a)}{s^2}$$

See Bode Plot.

Root Locus. When heavily damped, zero is always to the right of the poles

Closed Loop Bode Plot:

$$\log \left| \frac{\theta_o}{\theta_{in}} \right|$$



need  $\zeta > 4$  to meet jitter transfer spec of 0.1 dB

Here is where DLL is more effective.

No jitter peaking by nature -  
single pole CL TF

But, also requires a frequency reference with exact frequency.

Combined PLL/DLL systems can do this.

see Lee and Bulzacchelli,  
JSSC, 27, pp 1736-1746, Dec 1992.

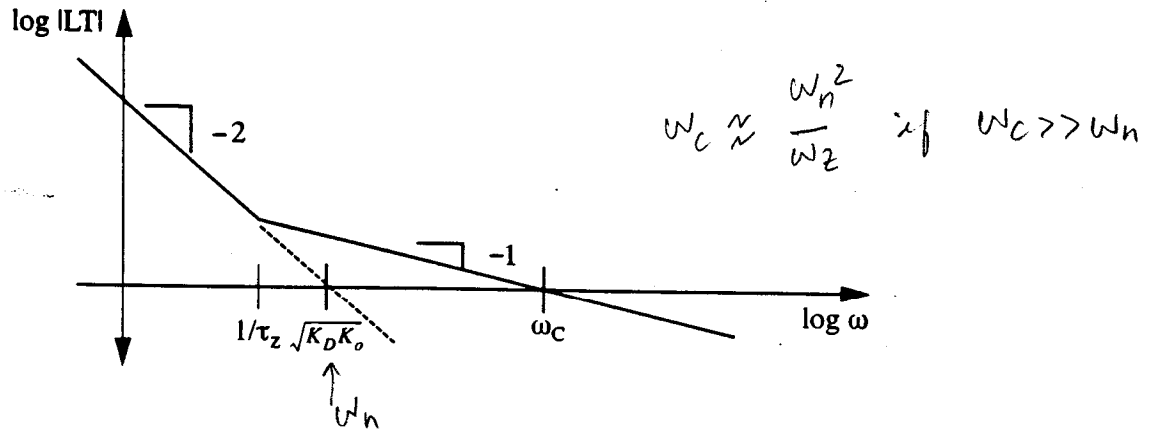


FIGURE 15.4. Loop transmission of second-order PLL.

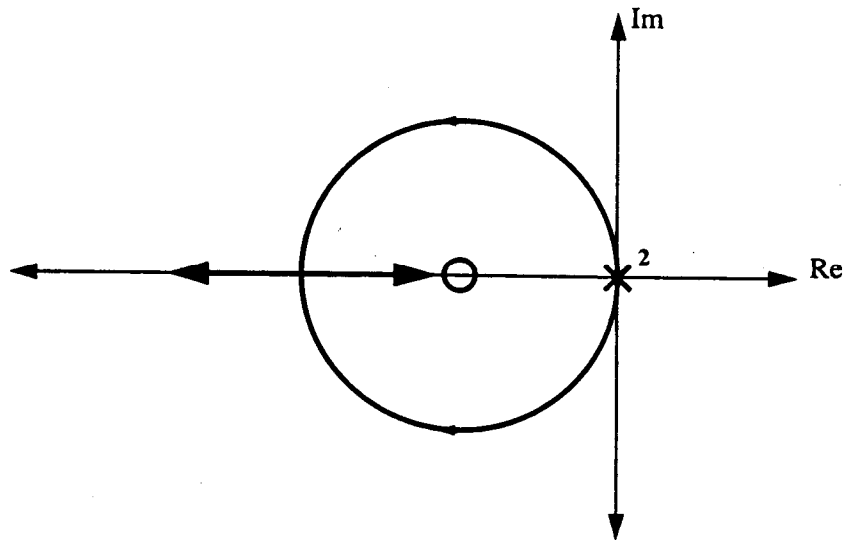
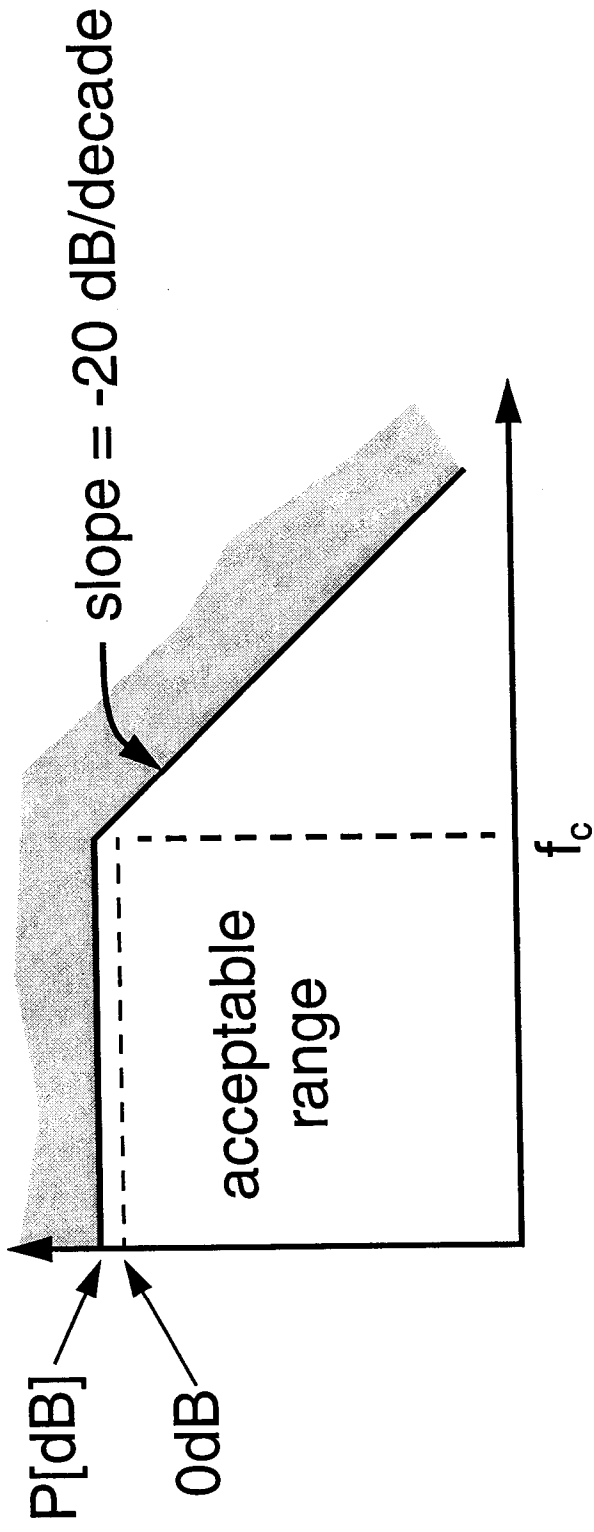


FIGURE 15.5. Root locus of second-order PLL.

T.H.Lee, Design of CMOS Radio Frequency ICs, 2nd Ed., Cambridge Press, 2004.

# Jitter Transfer Specification



Data Rate	$f_c$ [kHz]	P[dB]
155 Mb	130	0.1
622 Mb	500	0.1
2.488 Gb	2000	0.1

This specification is intended to control jitter peaking in long repeater chains

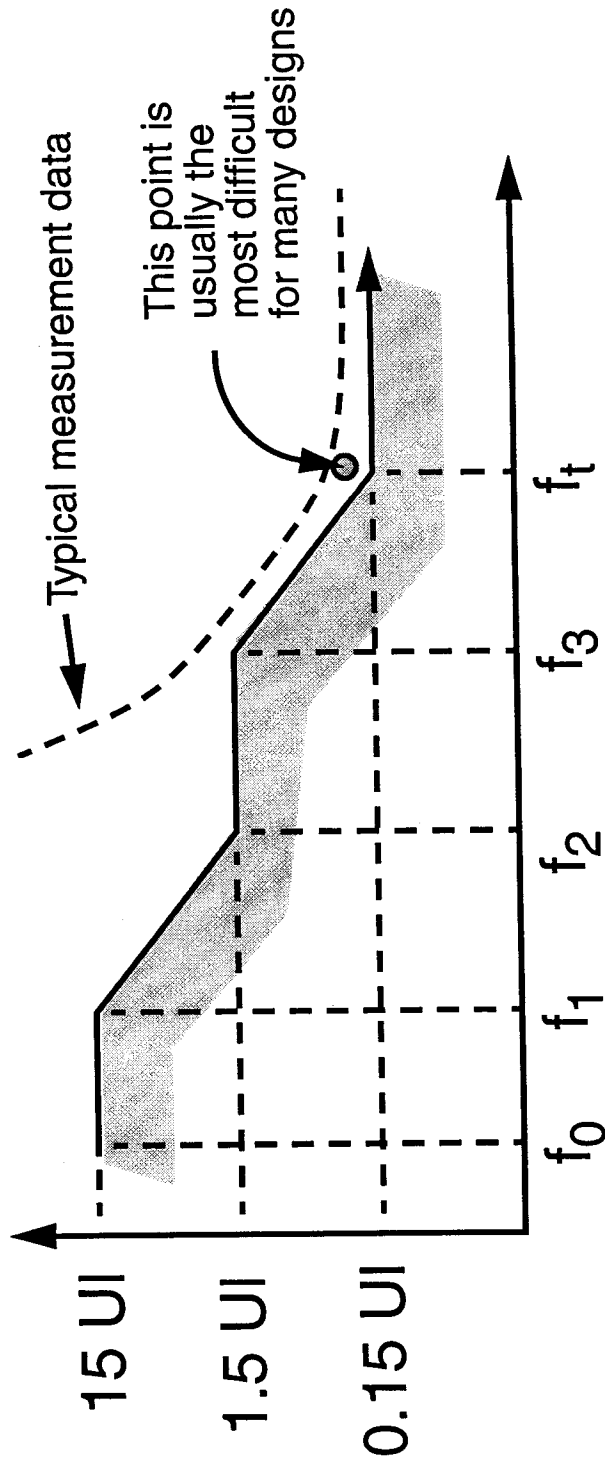
Walker, op.cit.

## Jitter Tolerance.

ability of PLL to track jitter on incoming data signal. Phase modulation

PLL must follow this jitter to enable accurate retiming.

# SONET Jitter Tolerance Mask



Data Rate	$f_0$ [Hz]	$f_1$ [Hz]	$f_2$ [Hz]	$f_3$ [kHz]	$f_t$ [kHz]
OC-3	10	30	300	6.5	65
OC-12	10	30	300	25	250
OC-48	10	600	6000	100	1000
OC-192	10	2400	24000	400	4000