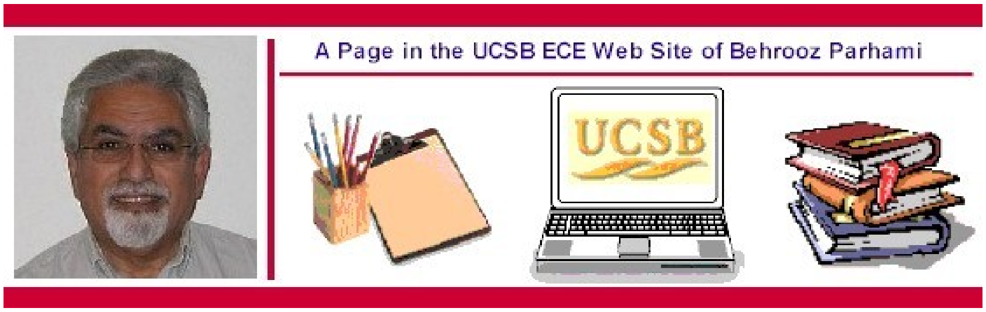

ECE 252B: History and Previous Offerings

Behrooz Parhami: 2009/01/06 || E-mail: parhami at ece.ucsb.edu || Other contact info at: [Bottom of this page]
Go up to: [B. Parhami's course syllabi] or [his home page]

Background and history of ECE 252B

Professor Parhami took over the teaching of ECE 252B from the late Dr. James Howard in the winter quarter of 1989. By covering sequential machines, computer arithmetic, and advanced microprocessor-based design, the graduate course sequence ECE 252A/B/C was meant to provide the students with a firm foundation in the theories and techniques of advanced digital design. During the first few offerings of ECE 252B, Professor Parhami gradually modified the content to increase both its coverage and research orientation (ECE 252A and 252C later underwent similar transformations by Professor Kwang-Ting Cheng and Professor Parhami, respectively). In 2000, based on a decade of experience in teaching this course, Professor Parhami published a graduate-level textbook (Oxford University Press), which is being used at many universities worldwide. The second edition of this textbook is scheduled to appear in mid to late 2009. The link *Computer Arithmetic: Algorithms and Hardware Designs* takes you to the book's Web page which contains brief and complete tables of contents, presentation material, and a list of known errors.

Special note: Web links in the following descriptions may be out of date. Please refer to the most recent offering of ECE 252B for up-to-date information.

Link to the most recent offering of ECE 252B: Computer Arithmetic

Previous offerings of ECE 252B

- Spring Quarter 2008
- Spring Quarter 2007
- Fall Quarter 2005 (course cancelled)
- Winter Quarter 2005
- Fall Quarter 2003
- Fall Quarter 2002
- Fall Quarter 2001
- Winter Quarter 2001
- Winter Quarter 2000

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ECE 252B: Spring Quarter 2008 offering

This area is reserved for important course announcements: **2008/05/30: Presentations for Parts V and VI have now been updated for spring 2008 in the textbook's website. Except for grade stats for HW4 and the final exam, to be posted as they become available, all information on this page is now final and will not be further updated.**

2008/05/25: HW4 (last one for the course) has been posted below. Please also see the posted notes on homework solutions.

2008/05/19: Updated presentations for part IV of the text have been posted.

2008/05/11: HW3 has been posted below. The midterm exam grades have a mean of 62 and a median of 65. The grade list is as follows -- 39 42 45 47 48 57 58 62 62 67 68 69 70 73 74 76 81 86

2008/04/22: HW2 has been posted below. Updated presentation for part III of the textbook will be posted by April 30.

0008/04/07: HW1 and a preliminary list of research topics have been posted below. Updated presentation for part I of the textbook is now available

on the book's website.

2008/03/20: Welcome to the ECE 252B website for spring 2008. During the quarter, you will need to consult two websites associated with the course. This website is the "course website." There is also a website for our textbook, where downloadable presentations and information on discovered errors can be found. You can reach the latter website by clicking on the textbook link under "References" below.

Course:	ECE 252B – Computer Arithmetic, University of California, Santa Barbara, Spring 2008, Enrollment Code 10736
Catalog entry:	**252B. Computer Arithmetic. (4) PARHAMI.** *Prerequisites: ECE 152A-B. Lecture, 4 hours.* Standard and unconventional number representations. Design of fast two-operand and multioperand adders. High-speed multiplication and division algorithms. Floating-point numbers, algorithms, and errors. Hardware algorithms for function evaluation. Pipelined, digit-serial and fault-tolerant arithmetic processors.
Instructor:	Behrooz Parhami, Room 5155 HFH (Engineering I), Phone 805-893-3211, parhami at ece.ucsb.edu
Meetings:	TR 12:00-1:30, Room 1431 Phelps Hall
Consultation:	Open office hours, held in Room 5155 HFH (Engineering I) – T 9:00-10:30, R 10:00-11:30
Motivation:	Computer arithmetic is a subfield of digital computer organization. It deals with the hardware realization of arithmetic functions to support various computer architectures as well as with arithmetic algorithms for firmware/software implementation. A major thrust of digital computer arithmetic is the design of hardware algorithms and circuits to enhance the speed of various numeric operations. Thus much of what is presented in this course complements the *architectural* and *algorithmic* speedup techniques covered as part of the advanced computer architecture (ECE 254A/B/C) sequence.
Prerequisites:	Familiarity with logic design and switching theory as well as fundamentals of digital system design (ECE 152A and ECE 152B or equivalents).
References:	***Required textbook*** *(expected at the UCSB bookstore in late March 2008)* Parhami, B., *Computer Arithmetic: Algorithms and Hardware Designs*, Oxford, 2000 (click on the title of the textbook to access its website). The Bookstore has been instructed to order the 4th or a later printing of the book which has all known typos and other errors corrected and contains some additional material. If you buy a used copy of the third or earlier printing, please make sure you correct the errors using the errata provided in the book's website (accessible via the link above). The price at the Bookstore is $125.00 for new books and $93.75 for used copies. ***Other useful books, not required*** Deschamps/Bioul/Sutter, *Synthesis of Arithmetic Circuits: . . .* , 2006 (TK7895.A65D47). Ercegovac/Lang, *Digital Arithmetic*, Morgan Kaufmann, 2004. Swartzlander, *Computer Arithmetic*, vols. 1-2, IEEE Computer Soc. Press (QA76.6.C633). Koren, *Computer Arithmetic Algorithms*, 2nd ed., A K Peters, 2002. Omondi, *Computer Arithmetic Systems: . . .* , Prentice-Hall, 1994 (QA76.9.C62O46). Ercegovac/Lang, *Division and Square Root: . . .* , Kluwer, 1994 (QA76.9.C62E73). Oklobdzija, *High-Performance System Design*, IEEE Press, 1999 (TK7871.99.M44037). Waser/Flynn, *Intro. to Arithmetic for Digital Systems Designers* (TK7895.A65W37.1982). Knuth, *The Art of Computer Programming: Seminumerical Algorithms* (QA76.6.K64 vol 2). Hwang, *Computer Arithmetic: Principles, Architecture, and Design* (TK7888.3.H9). Kulisch/Miranker, *Computer Arithmetic in Theory and Practice* (QA162.K84). ***Research Sources*** *Proc. Symp. Computer Arithmetic* (1969, 72, 75 78, and odd years since 1981). *IEEE Trans. Computers*, particularly special issues on computer arithmetic (8/70, 6/73, 7/77, 4/83, 8/90, 8/92, 8/94, 7/98, 7/00, 3/05, 8/08?). ***Electronic Resources at UCSB*** http://www.library.ucsb.edu/eresources/databases/ (electronic journals, collections, etc.) http://www.library.ucsb.edu/subjects/engineering/ece.html (research guide in ECE)
Evaluation:	Students will be evaluated based on the following three components with the given weights: 20% -- Homework (see the course calendar for general requirements and schedule). 30% -- Closed-book midterm exam (see the course calendar for date and coverage). 50% -- Open-book final exam (see the course calendar for date, time, and coverage)
Research:	An optional research paper may be substituted for the final exam. A student interested in this option, will review a subfield of computer arithmetic or do original research on a selected topic. A list of research topics is provided below; however, please feel free to propose your own topic for approval. A publishable report earns an "A" for the course, regardless of homework and midterm grades. See the "deadlines" column in course calendar for schedule and due dates.

Calendar:

Course lectures, homework assignments, and exams have been scheduled as follows. This schedule will be strictly observed. Textbook chapters are specified for lectures and exams. It is a good idea to look over the specified material *before* each lecture if possible.

Day & Date	Chap	Lecture Topic	Deadlines and Notes
Tues. 4/1	1-2	Numbers and signed values	This is a serious lecture -- Really!
Thur. 4/3	3-4	Redundant and residue representations	

Tues. 4/8	5	Basic addition and counting	HW#1 posted (chapters 1-7)
Thur. 4/10	6	Carry-lookahead adders	Research topic defined
Tues. 4/15	7	Variations in fast adders	
Thur. 4/17	8	Multioperand addition	HW#1 due
Tues. 4/22	9	Basic multiplication schemes	HW#2 posted (chapters 8-11)
Thur. 4/24	10	High-radix multipliers	
Tues. 4/29	11	Tree and array multipliers	Preliminary references due
Thur. 5/1	12	Variations in multipliers	HW#2 due
Tues. 5/6	1-11	Midterm exam: closed book (12-2 PM)	Held in class; note extended time
Thur. 5/8	13	Basic division schemes	
Tues. 5/13	14	High-radix division	HW#3 posted (chapters 12-16)
Thur. 5/15	15	Variations in dividers	Paper title and references due
Tues. 5/20	16	Division by convergence	
Thur. 5/22	17-18	Floating-point numbers and operations	HW#3 due
Tues. 5/27	19-20	Errors, precision, and certifiability	HW#4 posted (chapters 17-22)
Thur. 5/29	21	Square-rooting methods	Paper abstract and outline due
Tues. 6/3	22	CORDIC algorithms	
Thur. 6/5	23-24	Other topics in function evaluation	HW#4 due
Mon. 6/9	1-24	Final exam: open book (12-3 PM)	Held in class, Complete paper due

Homework: General Requirements

Solutions should be brought to class on the due date and handed in before the lecture begins.
Late homework will not be accepted, so plan to start work on your assignments early.
Use a cover page that includes your name, course and assignment number for your solutions.
Staple the sheets and write your name on top of every sheet in case sheets are separated.
Although some cooperation is permitted, direct copying will have severe consequences.

ECE 252B s2008, Homework #1: Number Systems and Addition (chaps. 1-7, due Thur. April 17)

Do the following problem from the textbook or listed below: 1.I [20 pts.], 3.9a [15 pts.], 5.3 [10 pts.], 5.8 [15 pts.], 6.3 [15 pts.], 7.7 [25 pts]

Problem 1.I (Fixed-radix positional number systems) Let $N_{k,r}$ be an integer whose representation in radix r consists of k digits of 1, that is, $N_{k,r} = (1\ 1\ .\ .\ .\ 1)_r$, where the number of 1 digits is k. (a) Show the 2k-bit radix-2 representation of the square of $N_{k,2}$. (b) Prove that except for $N_{1,10}$, no $N_{i,10}$ is a perfect square. (c) Show that $N_{i,10}$ divides $N_{j,10}$ iff i divides j.

ECE 252B s2008, Homework #2: Multioperand Addition and Multiplication (chaps. 8-11, due Thur. May 1)

Do the following problem from the textbook: 8.7 [15 pts.], 8.12 [20 pts.], 9.9a [10 pts.], 9.12 [20 pts.], 10.1 [15 pts.], 11.10 [20 pts]

ECE 252B s2008, Homework #3: Multiplication and Division (chaps. 12-16, due Thur. May 22)

Do the following problem from the textbook: 12.13 [15 pts.], 12.15 [15 pts.], 13.5a [10 pts.], 13.12a [15 pts.], 14.8 [20 pts.], 15.5 [10 pts], 16.12 [15 pts.]

ECE 252B s2008, Homework #4: Floating-Point and Function Evaluation (chaps. 17-22, due Thur. June 5)

Do the following problem from the textbook: 17.7 [15 pts.], 18.6bc [15 pts.], 19.7 [20 pts.], 20.2 [10 pts.], 21.2b [20 pts.], 22.2c [20 pts.]

ECE 252B s2008, Notes on homework solutions and other topics

For HW2, Problem 12.13c, the suggested substitutions (essentially the same as for the Baugh-Wooley method) applied to the simplified bit-matrix of part b yields the following in columns 4-6 (column 7 is ignored because the square is always positive, and columns 0-3 do not change). Column 4: x3, x2x1', x3x0'. Column 5: x2x1, x3x1'. Column 6: x3, x3x2'. The required circuit consists of a 3-bit CSA and a 4-bit adder (or, equivalently, a 4-bit CSA and a 3-bit adder). Ignore the last sentence in the solution handout, as it is incorrect.

For HW2, Problem 13.12, the term 1 + 2^(-36) and the associated shift-add step are not needed, given 32 bits of precision.

ECE 252B s2008, Possible Research Topics

a

1. Mod-(2 + 1) Number Representations and Arithmetic

2. A Survey of Hardware Multipliers in Microprocessors

3. Radix-16 SRT Division in Intel's New Penryn Processors

4. Augmenting FPGAs for Faster Arithmetic Operations

5. Cube Roots: Hardware Algorithms and Applications

6. Accurate Summation of Sets of Floating-Point Numbers

7. Function Evaluation by Piecewise Linear Approximation

8. Smaller Lookup Tables via Nonuniform Segmentation

9. Arithmetic in the European Logarithmic Microprocessor

10. Arithmetic in IBM's Blue Gene/L Parallel Supercomputer

ECE 252B s2008, Midterm Exam Preparation

Textbook sections that are not required for the closed-book midterm exam: 3.5, 3.4, 3.6, 4.4, 4.5, 4.6, 6.3, 7.2, 10.5 (some of these may be included in the final exam, which will be open-book)

ECE 252B s2008, Final Exam Preparation

The final exam will include Chapters 1-24, emphasizing the topics covered after the midterm. The following sections are excluded: 3.5 (pp. 45-48), 3.6, 4.4, 4.5, 4.6, 7.2, 10.5, 15.4, 15.6, 19.4, 19.5, 19.6, 20.2, 21.4, 21.6, 22.6, 23.4, 24.4-24.6. All other sections are included, even if they were not discussed in class.

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ECE 252B: Spring Quarter 2007 offering

This area is reserved for important course announcements: The textbook's website now has updated presentations for all material covered in class (Parts I-VI). This course page is now final and will not be further updated during this quarter

Course:	ECE 252B – Computer Arithmetic, University of California, Santa Barbara, Spring 2007, Enrollment Code 53470.
Catalog entry:	**252B. Computer Arithmetic.** **(4) PARHAMI.** *Prerequisites: ECE 152A-B. Lecture, 4 hours.* Standard and unconventional number representations. Design of fast two-operand and multioperand adders. High-speed multiplication and division algorithms. Floating-point numbers, algorithms, and errors. Hardware algorithms for function evaluation. Pipelined, digit-serial and fault-tolerant arithmetic processors. (F)
Instructor:	Behrooz Parhami, Room 5155 HFH (Engineering I), Phone 805-893-3211, parhami at ece.ucsb.edu
Meetings:	TR 12:00-1:30, Room 1431 Phelps Hall
Consultation:	Open office hours, held in Room 5155 HFH (Engineering I) – T 2:00-3:30, R 10:00-11:30
Motivation:	Computer arithmetic is a subfield of digital computer organization. It deals with the hardware realization of arithmetic functions to support various computer architectures as well as with arithmetic algorithms for firmware/software implementation. A major thrust of digital computer arithmetic is the design of hardware algorithms and circuits to enhance the speed of various numeric operations. Thus much of what is presented in this course complements the *architectural* and *algorithmic* speedup techniques covered as part of the advanced computer architecture (ECE 254A/B/C) sequence.
Prerequisites:	Familiarity with logic design and switching theory as well as fundamentals of digital system design (ECE 152A and ECE 152B or equivalents).
References:	***Required textbook*** *(expected at the UCSB bookstore in late March 2007)* Parhami, B., *Computer Arithmetic: Algorithms and Hardware Designs*, Oxford, 2000. The Bookstore has been instructed to order the 4th or a later printing of the book which has all known typos and other errors corrected and contains some additional material. If you buy a used copy of the third or earlier printing, please make sure you correct the errors using the errata provided in the book's Web page (accessible via the link above). ***Other useful books, not required*** Ercegovac/Lang, *Digital Arithmetic*, Morgan Kaufmann, 2004. Swartzlander, *Computer Arithmetic*, vols. 1-2, IEEE Computer Soc. Press (QA76.6.C633). Koren, *Computer Arithmetic Algorithms*, 2nd ed., A K Peters, 2002. Omondi, *Computer Arithmetic Systems: . . .* , Prentice-Hall, 1994 (QA76.9.C62O46). Ercegovac/Lang, *Division and Square Root: . . .* , Kluwer, 1994 (QA76.9.C62E73). Oklobdzija, *High-Performance System Design*, IEEE Press, 1999 (TK7871.99.M44037).

Waser/Flynn, *Intro. to Arithmetic for Digital Systems Designers* (TK7895.A65W37.1982).
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Hwang, *Computer Arithmetic: Principles, Architecture, and Design* (TK7888.3.H9).
Kulisch/Miranker, *Computer Arithmetic in Theory and Practice* (QA162.K84).
Research Sources
Proc. Symp. Computer Arithmetic (1969, 72, 75 78, and odd years since 1981).
IEEE Trans. Computers, particularly special issues on computer arithmetic (8/70, 6/73, 7/77, 4/83, 8/90, 8/92, 8/94, 7/98, 7/00, 3/05).
Electronic Resources at UCSB
http://www.library.ucsb.edu/eresources/databases/ (electronic journals, collections, etc.)
http://www.library.ucsb.edu/subjects/engineering/ece.html (research guide in ECE)

Evaluation: Students will be evaluated based on the following three components with the given weights:

20% -- Homework (see the course calendar for general requirements and schedule).

30% -- Closed-book midterm exam (see the course calendar for date and coverage).

50% -- Open-book final exam (see the course calendar for date, time, and coverage)

Research: An optional research paper may be substituted for the final exam. A student interested in this option, will review a subfield of computer arithmetic or do original research on a selected topic. A list of research topics is provided below; however, please feel free to propose your own topic for approval. A publishable report earns an “A” for the course, regardless of homework and midterm grades. See the “deadlines” column in course calendar for schedule and due dates.

Calendar:

Course lectures, homework assignments, and exams have been scheduled as follows. This schedule will be strictly observed. Textbook chapters are specified for lectures and exams. It is a good idea to look over the specified material *before* each lecture if possible.

Day & Date	Chap	Lecture Topic	Deadlines and Notes
Tues. 4/3	1-2	Numbers and signed values	
Thur. 4/5	3-4	Redundant and residue representations	
Tues. 4/10	5	Basic addition and counting	HW#1 posted (chapters 1-7)
Thur. 4/12	6	Carry-lookahead adders	Research topic defined
Tues. 4/17	7	Variations in fast adders	
Thur. 4/19	8	Multioperand addition	HW#1 due
Tues. 4/24	9	Basic multiplication schemes	HW#2 posted (chapters 8-11)
Thur. 4/26	10	High-radix multipliers	
Tues. 5/1	11	Tree and array multipliers	Preliminary references due
Thur. 5/3	12	Variations in multipliers	HW#2 due
Tues. 5/8	1-11	Midterm exam: closed book (12-2 PM)	Held in class; note extended time
Thur. 5/10	13	Basic division schemes	
Tues. 5/15	14	High-radix division	HW#3 posted (chapters 12-16)
Thur. 5/17	15	Variations in dividers	Paper title and references due
Tues. 5/22	16	Division by convergence	
Thur. 5/24	17-18	Floating-point numbers and operations	HW#3 due
Tues. 5/29	19-20	Errors, precision, and certifiability	HW#4 posted (chapters 17-22)
Thur. 5/31	21	Square-rooting methods	Paper abstract and outline due
Tues. 6/5	22	CORDIC algorithms	
Thur. 6/7	23-24	Other topics in function evaluation	HW#4 due
Mon. 6/11	1-24	Final exam: open book (12-3 PM)	Held in class, Complete paper due

Homework: General Requirements

Solutions should be brought to class on the due date and handed in before the lecture begins.
Late homework will not be accepted, so plan to start work on your assignments early.
Use a cover page that includes your name, course and assignment number for your solutions.
Staple the sheets and write your name on top of every sheet in case sheets are separated.
Although some cooperation is permitted, direct copying will have severe consequences.

ECE 252B s2007, Homework #1: Number Systems and Addition (chaps. 1-7, due Thur. April 19)

Do the following problems from the textbook or defined below: 1.5 (15 pts.), 2.A (15 pts.), 5.11 (25 pts.), 6.1 (10 pts.), 6.8 (20 pts.), 7.16 (15 pts.)

Problem 2.A Biased number representation: Consider radix-2 fractional numbers in the range [–1, 1), represented with a bias of 1. (a) Develop an addition algorithm for such biased numbers. (b) Show that sign change for such numbers is identical to 2's complementation. (c) Use the results of parts a and b to design an adder/subtractor for such numbers.

ECE 252B s2007, Homework #2: Multioperand Addition and Multiplication (chaps. 8-11, due Thur. May 3)

Do the following problems from the textbook or defined below: 8.4 (20 pts.), 9.1 (5 pts.), 9.6 (15 pts.), 10.6 (30 pts.), 11.5 (20 pts.), 11.17 (10 pts.)

ECE 252B s2007, Homework #3: Multiplication and Division (chaps. 12-16, due Thur. May 24)

Do the following problems from the textbook or defined below: 12.14 (15 pts.), 13.6b (15 pts.), 13.12a (15 pts.), 14.14 (25 pts.), 15.12abc (15 pts.), 16.4 (15 pts.)

ECE 252B s2007, Homework #4: Floating-Point and Function Evaluation (chap. 17-22, due Thur. June 7)

Do the following problems from the textbook or defined below: 17.2 (15 pts.), 17.14 (20 pts.), 18.18 (15 pts.), 19.2 (15 pts.), 21.15 (15 pts.), 22.4c (20 pts.)

ECE 252B s2007, Notes on homework solutions and other topics

None at this time.

ECE 252B s2007, Possible Research Topics

List of topics will be posted here if some students indicate interest in the research option in lieu of final exam.

ECE 252B s2007, Midterm Exam Preparation

Textbook sections that are not required for the closed-book midterm exam: 3.5, 3.6, 4.4, 4.5, 4.6, 6.3, 7.2, 10.5 (some of these may be included in the final exam, which will be open-book)

ECE 252B s2007, Final Exam Preparation

The final exam will include Chapters 1-24, emphasizing the topics covered after the midterm. The following sections are excluded: 3.5 (pp. 45-48), 3.6, 4.4, 4.5, 4.6, 7.2, 10.5, 15.4, 15.6, 19.4, 19.5, 19.6, 20.2, 21.4, 21.1-21.4, 21.6, 22.6, 23.4, 24.4-24.6. All other sections are included, even if they were not discussed in class.

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ECE 252B: Fall Quarter 2005 offering

This area is reserved for important course announcements: The course was cancelled for the fall 2005 quarter; its next offering will be in Spring 2007.

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ECE 252B: Winter Quarter 2005 offering

This area is reserved for important course announcements: A new special issue of *IEEE Trans. Computers* has just appeared (March 2005), leading to an update in the References section. Some electronic resources have also been added to the References section. Course grades were assigned and reported on 2005/03/15.

Course:	ECE 252B – Computer Arithmetic, University of California, Santa Barbara, Winter 2005, Enrollment Code 54320. This has been a fall-quarter course in the past few years and may revert to that schedule next year.
Catalog entry:	**252B. Computer Arithmetic. (4) PARHAMI.** *Prerequisites: ECE 152A-B. Lecture, 4 hours.* Standard and unconventional number representations. Design of fast two-operand and multioperand adders. High-speed multiplication and division algorithms. Floating-point numbers, algorithms, and errors. Hardware algorithms for function evaluation. Pipelined, digit-serial and fault-tolerant arithmetic processors. (F)
Instructor:	Behrooz Parhami, Room 5155 Engineering I, Phone 805-893-3211, parhami at ece.ucsb.edu
Meetings:	MW 10:00-11:30, Room 3519 Phelps Hall

Consultation:	Open office hours, held in Room 5155 Engineering I – M 11:40-1:00, W 12:30-2:00
Motivation:	Computer arithmetic is a subfield of digital computer organization. It deals with the hardware realization of arithmetic functions to support various computer architectures as well as with arithmetic algorithms for firmware/software implementation. A major thrust of digital computer arithmetic is the design of hardware algorithms and circuits to enhance the speed of various numeric operations. Thus much of what is presented in this course complements the *architectural* and *algorithmic* speedup techniques covered as part of the advanced computer architecture (ECE 254A/B/C) sequence.
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Evaluation:	Students will be evaluated based on the following three components with the given weights: 20% -- Homework (see the course calendar for general requirements and schedule). 30% -- Closed-book midterm exam (see the course calendar for date and coverage). 50% -- Open-book final exam (see the course calendar for date, time, and coverage)
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Calendar:

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Day & Date	Chap	Lecture Topic	Deadlines and Notes
Mon. 1/3	1-2	Numbers and signed values	
Wed. 1/5	3-4	Redundant and residue systems	
Mon. 1/10	5-6	Basic and carry-lookahead addition	HW#1 posted (chapters 1-7)
Wed. 1/12	6-7	Other topics in fast addition	Research topic defined
Mon. 1/17		Dr. King's birthday -- no lecture	
Wed. 1/19	8	Multioperand addition	HW#1 due
Mon. 1/24	9	Basic multiplication schemes	HW#2 posted (chapters 8-12)
Wed. 1/26	10-11	High-radix and tree multipliers	
Mon. 1/31	11-12	Other topics in multiplication	Preliminary references due
Wed. 2/2		Review and make-up	HW#2 due
Mon. 2/7	1-12	Midterm exam: closed book (10-12 AM)	Held in class; note extended time
Wed. 2/9	13	Basic division schemes	HW#3 posted (chapters 13-16)
Mon. 2/14	14-15	High-radix division	

Wed. 2/16	15-16	Other topics in division	Paper title and references due
Mon. 2/21		President's day holiday -- no lecture	
Wed. 2/23	17-18	Floating-point numbers and operations	HW#3 due
Mon. 2/28	19-20	Errors, precision, and certifiability	HW#4 posted (chapters 17-22)
Wed. 3/2	21	Square-rooting methods	Paper abstract and outline due
Mon. 3/7	22	CORDIC algorithms	
Wed. 3/9	23-24	Other topics in function evaluation	HW#4 due
Mon. 3/14	1-24	Final exam: open book (8:30-11 AM)	Held in class, Complete paper due

Homework: General Requirements

Solutions should be brought to class on the due date and handed in before the lecture begins.
Late homework will not be accepted, so plan to start work on your assignments early.
Use a cover page that includes your name, course and assignment number for your solutions.
Staple the sheets and write your name on top of every sheet in case sheets are separated.
Although some cooperation is permitted, direct copying will have severe consequences.

ECE 252B w2005, Course Grade Stats

Item	Out of	Max	Mean	Median	Min	SD
HW #1	100	80	62	61	47	n/a
HW #2	100	82	67	74	37	n/a
HW #3	100	96	82	76	74	n/a
HW #4	100	86	79	82	66	n/a
Midterm	100	82	57	55	36	n/a
Final/Paper	100	83	70	69	59	n/a
Course*	100	83	65	60	58	n/a
Course	A (4.0)	A+	3.3	3.0	B	n/a

* Course = 20%(HW total / 4) + 30%(Midterm) + 50%(Final or Paper)

ECE 252B w2005, Homework #1: Number Systems and Addition (chap. 1-7, due Wed. Jan. 19)

Do the following problems from the textbook or defined below: 2.1 [20 pts.], 3.11 [15 pts.], 5.A [15 pts.], 6.F [10 pts], 7.13 [20 pts.], 7.15 [20 pts.]

Problem 5.A Binary adders as versatile building blocks -- Show how to use a 4-bit binary adder as: **(a)** A multiply-by-3 circuit for a 4-bit unsigned number. **(b)** Two independent 3-input majority circuits implementing 2-out-of-3 voting.

Problem 6.F Implementing the carry operator -- Show that the carry operator of Fig. 6.6 can be implemented by using *g* = (*g*¢ Ú *g*²)(*p*² Ú *g*²), thereby making all signals for *p* and *g* go through two levels of logic using a NOT-NOR or NOR-NOR implementation.

ECE 252B w2005, Homework #2: Multioperand Addition and Multiplication (chap. 8-12, due Wed. Feb. 2)

Do the following problems from the textbook or defined below: 8.15 [25 pts.], 9.4c [20 pts], 9.15 [15 pts.], 10.5 [10 pts.], 11.E [15 pts.], 12.16 [15 pts.]

Problem 11.E Unsigned/2's-complement tree multiplier – Most processors allow both unsigned and signed numbers as operands in their integer arithmetic units. Discuss the design of a *k* ´ *k* tree multiplier that can act as an unsigned multiplier (for *t* = 0) or 2's-complement multiplier (for *t* = 1), where *t* is a control signal.

ECE 252B w2005, Homework #3: Division (chap. 13-16, due Wed. Feb. 23)

Do the following problems from the textbook or defined below: 13.7b [20 pts.], 13.D [20 pts.], 14.2d [20 pts], 15.13abc [20 pts.], 16.A [20 pts.]

Problem 13.D Sequential division – Perform the unsigned binary division **.** 1100 1100 1100 / **.** 1001 11 by means of the restoring algorithm. *Hint:* Watch for overflow.

Problem 16.A Sequential vs convergence division – Suppose multiplication and addition take 5 and 1 time units, respectively, and that all support and control functions (counting, conditionals, register transfers, etc.) take negligible time due to overlapped processing. Be brief and state all your assumptions clearly. **(a)** Express the time needed for simple binary restoring division as a function of the word width *k*. **(b)** Express the time needed for division by repeated multiplications (without an initial table lookup or other speedup methods) as a function of *k*. **(c)** Compare the results of parts a and b. Comment on the speed/cost tradeoffs for different word widths.

ECE 252B w2005, Homework #4: Floating-Point and Function Evaluation (chap. 17-22, due Wed. Mar. 9)

Do the following problems from the textbook or defined below: 17.11 [20 pts.], 18.F [20 pts.], 20.3 [20 pts.], 21.2a [20 pts.], 22.2ab [20 pts.]

Problem 18.F Monotonicity in floating-point arithmetic – This problem is attributed to W. Kahan. Consider a computer that performs floating-point multiplication by truncating (rather than rounding) the exact 2*p*-digit product of *p*-digit normalized fractional significands to *p* digits; that is, it either does not develop the lower *p* digits of the exact product, or simply drops them. **(a)** Show that, for a radix *r* greater than 2, this causes the monotonicity of multiplication to be violated (i.e., there exist positive floating-point numbers *a*, *b*, and *c* such that $a < b$ but a ´$_{fp}$ $c > b$ ´$_{fp}$ c). *Hint:* when x ´$_{fp}$ $y < 1/r$, postnormalization causes the least significant digit of the final product to be 0. **(b)** Show that multiplication remains monotonic in radix 2 (i.e., a £ b implies a ´$_{fp}$ c £ b ´$_{fp}$ c).

ECE 252B w2005, Notes on homework solutions and other topics

None.

ECE 252B w2005, Possible Research Topics

Given that no student has chosen the research path in lieu of the final exam, this section will not be updated for the current quarter.

ECE 252B w2005, Midterm Exam Preparation

Textbook sections that are not required for the closed-book midterm exam: 3.5, 3.6, 4.4, 4.5, 4.6, 6.3, 7.2, 10.5 (some of these may be included in the final exam, which will be open-book)

ECE 252B w2005, Final Exam Preparation

The final exam will include Chapters 1-24, emphasizing the topics covered after the midterm. The following sections are excluded: 3.5 (pp. 45-48), 3.6, 4.4, 4.5, 4.6, 7.2, 10.5, 15.4, 15.6, 19.4, 19.5, 19.6, 20.2, 21.4, 21.6, 22.6, 24.6. All other sections are included, even if they were not discussed in class.

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ECE 252B: Fall Quarter 2003 offering

Course:	ECE 252B – Computer Arithmetic, University of California, Santa Barbara, Fall 2003, Enrollment Code 11106
Catalog entry:	**252B. Computer Arithmetic.** **(4) PARHAMI.** *Prerequisites: ECE 152A-B. Lecture, 4 hours.* Standard and unconventional number representations. Design of fast two-operand and multioperand adders. High-speed multiplication and division algorithms. Floating-point numbers, algorithms, and errors. Hardware algorithms for function evaluation. Pipelined, digit-serial and fault-tolerant arithmetic processors. (F)
Instructor:	Behrooz Parhami, Room 5155 Engineering I, Phone 805-893-3211, parhami at ece.ucsb.edu
Meetings:	MW 4:00-5:30 PM, Room 1425 Phelps Hall
Consultation:	Open office hours, held in Room 5155 Engineering I – M 11:00-12:30, W 12:30-2:00
Motivation:	Computer arithmetic is a subfield of digital computer organization. It deals with the hardware realization of arithmetic functions to support various computer architectures as well as with arithmetic algorithms for firmware/software implementation. A major thrust of digital computer arithmetic is the design of hardware algorithms and circuits to enhance the speed of various numeric operations. Thus much of what is presented in this course complements the *architectural* and *algorithmic* speedup techniques covered as part of the advanced computer architecture (ECE 254A/B/C) sequence.
Prerequisites:	Familiarity with logic design and switching theory as well as fundamentals of digital system design (ECE 152A and ECE 152B or equivalents).
References:	***Required textbook*** *(available at the bookstore in September)* Parhami, B., *Computer Arithmetic: Algorithms and Hardware Designs*, Oxford, 2000. The Bookstore has been instructed to order the 4th printing of the book which has all known typos and other errors corrected and contains some additional material. If you buy a used copy of the third or earlier printing, please make sure you correct the errors using the errata provided in the book's Web page (accessible via the link above). ***Other useful books, not required*** Swartzlander, *Computer Arithmetic*, vols. 1-2, IEEE Computer Soc. Press (QA76.6.C633). Koren, *Computer Arithmetic Algorithms*, Prentice-Hall, 1993 (QA76.9.C62K67). Omondi, *Computer Arithmetic Systems: . . .* , Prentice-Hall, 1994 (QA76.9.C62O46). Ercegovac/Lang, *Digital Arithmetic*, Morgan Kaufmann, 2004 Ercegovac/Lang, *Division and Square Root: . . .* , Kluwer, 1994 (QA76.9.C62E73). Oklobdzija, *High-Performance System Design*, IEEE Press, 1999 (TK7871.99.M44037). Waser/Flynn, *Intro. to Arithmetic for Digital Systems Designers* (TK7895.A65W37.1982). Knuth, *The Art of Computer Programming: Seminumerical Algorithms* (QA76.6.K64 vol 2). Hwang, *Computer Arithmetic: Principles, Architecture, and Design* (TK7888.3.H9). Kulisch/Miranker, *Computer Arithmetic in Theory and Practice* (QA162.K84). ***Research Sources***

Proc. Symp. Computer Arithmetic (1969, 72, 75 78, and odd years since 1981).
IEEE Trans. Computers, particularly special issues on computer arithmetic (8/70, 6/73, 7/77, 4/83, 8/90, 8/92, 8/94, 7/98, 7/00).

Evaluation: Students will be evaluated based on the following three components with the given weights:

25% -- Homework (see the course calendar for general requirements and schedule).

25% -- Closed-book midterm exam (see the course calendar for date and coverage).

50% -- Open-book final exam (see the course calendar for date, time, and coverage)

Research: An optional research paper may be substituted for the final exam. A student interested in this option, will review a subfield of computer arithmetic or do original research on a selected topic. A list of research topics is provided below; however, please feel free to propose your own topic for approval. A publishable report earns an "A" for the course, regardless of homework and midterm grades. See the "deadlines" column in course calendar for schedule and due dates.

Calendar:

Course lectures, homework assignments, and exams have been scheduled as follows. This schedule will be strictly observed. Textbook chapters are specified for lectures and exams. It is a good idea to look over the specified material *before* each lecture if possible.

Day & Date	Chap	Lecture Topic	Deadlines and Notes
Mon. 9/22	1-2	Numbers and signed values	
Wed. 9/24	3	Redundant number representations	HW#1 posted (chapters 1-4)
Mon. 9/29	4	Residue number systems	
Wed. 10/1	5	Basic addition and counting	HW#2 posted (5-6), HW#1 due
Mon. 10/6	6	Carry-lookahead adders	Research topic defined
Wed. 10/8	7	Variations in fast adders	HW#3 posted (7-8), HW#2 due
Mon. 10/13	8	Multioperand addition	
Wed. 10/15	9	Basic multiplication schemes	HW#4 posted (9-10)
Mon. 10/20	10	High-radix multiplication	HW#3 due, Preliminary ref's due
Wed. 10/22	11	Tree and array multipliers	
Mon. 10/27	12	Variations in multipliers	HW#5 posted (11-12), HW#4 due
Wed. 10/29	1-10	Midterm exam: closed book (4-6 PM)	Note the extended time
Mon. 11/3	13	Basic division schemes	
Wed. 11/5	14	High-radix division	HW#6 posted (13-14)
Mon. 11/10	15	Variations in dividers	HW#5 due, Paper title & ref's due
Wed. 11/12	16	Convergence division	HW#7 posted (15-16)
Mon. 11/17	17-18	Floating-point numbers & operations	HW#6 due, Prelim. abstract due
Wed. 11/19	19-20	Errors, precision, & certifiability	HW#8 posted (17-20)
Mon. 11/24	21	Square-rooting methods	HW#7 due
Wed. 11/26		No lecture due to Thanksgiving	
Mon. 12/1	22	The CORDIC algorithms	HW#8 due
Wed. 12/3	23-24	Other topics in function evaluation	Complete paper due
Thur. 12/11	1-24	Final exam: open book (4:00-7:00 PM)	Place: our regular classroom

Homework: General Requirements

Solutions should be brought to class on the due date and handed in before the lecture begins.
Late homework will not be accepted, so plan to start work on your assignments early.
Use a cover page that includes your name, course and assignment number for your solutions.
Staple the sheets and write your name on top of every sheet in case sheets are separated.
Although some cooperation is permitted, direct copying will have severe consequences.

Course Grade Stats

Item	Out of	Max	Mean	Median	Min	SD
HW #1	100	86	56	56	37	17
HW #2	100	78	63	60	48	10

HW #3	100	92	78	82	58	12
HW #4	100	98	79	74	67	11
HW #5	100	83	61	61	46	13
HW #6	100	98	88	90	76	9
HW #7	100	82	57	60	23	24
HW #8	100	73	56	53	40	12
Midterm	100	80	71	70	62	7
Final/Paper	100	77	65	60	57	9
Course*	100	75	67	67	62	5
Course	A (4.0)	A	3.3	B+	B	0.41

* Course = 25%(HW total / 8) + 25%(Midterm) + 50%(Final or Paper)

ECE 252B f2003, Homework #1: Number Representation (chapters 1-4, due Wed. Oct. 1)

Do the following problems from the textbook: 1.3, 1.11, 2.13, 3.4a (part a only), 4.2

ECE 252B f2003, Homework #2: Basic & Carry-Lookahead Addition (chapters 5-6, due Wed. Oct. 8)

Do the following problems from the textbook: 5.6def, 5.10, 6.5, 6.12, 6.16

ECE 252B f2003, Homework #3: Other Addition Topics (chapters 7-8, due Mon. Oct. 20)

Do the following problems from the textbook or defined below: 7.6, 7.8, 7.C, 8.11, 8.B

Problem 7.C (Carry-skip versus carry-select adders): Compare the costs and delays of 16-bit single-level carry-skip and carry-select adders, each constructed from 4-bit adder blocks and additional logic as needed. Which design would you consider more cost-effective and why? State all your assumptions clearly.

Problem 8.B (carry-save adder trees): **a.** Show the widths of the four CSAs required to reduce six 32-bit unsigned binary numbers to two. **b.** Repeat part a, but assume that the six 32-bit unsigned numbers are the partial products of a 32-by-6 multiplication (i.e., they are not aligned at the LSBs but shifted to the left by 0, 1, 2, 3, 4, and 5 bits).

ECE 252B f2003, Homework #4: Basic & High-Radix Multiplication (chapters 9-10, due Mon. Oct. 27)

Do the following problems from the textbook: 9.3, 9.9b, 9.16, 10.8a, 10.9

ECE 252B f2003, Homework #5: Other Multiplication Topics (chapters 11-12, due Mon. Nov. 10)

Do the following problems from the textbook: 11.10, 11.12, 12.1, 12.9*

* Correction to Problem 12.9: Change $2k - 2j + 1$ to $2k - 2j - 1$

ECE 252B f2003, Homework #6: Basic & High-Radix Division (chapters 13-14, due Mon. Nov. 17)

Do the following problems from the textbook or defined below: 13.14, 13.A, 14.1cd, 14.5

Problem 13.A (sequential division): Perform the unsigned fractional binary division .0110 1101 / .1011 using the restoring and nonrestoring methods

ECE 252B f2003, Homework #7: Other Division Topics (chapters 15-16, due Mon. Nov. 24)

Do the following problems from the textbook or defined below: 15.2, 15.B, 16.7, 16.13

Problem 15.B (radix-8 division): Argue that the use of the quotient digit set [–6, 6] is preferable to the minimal digit set [–4, 4] in a radix-8 divider that forms the multiple $3d$ by inputting the two values $2d$ and d into a four-input carry-save adder tree that also receives the carry-save partial remainder as inputs.

ECE 252B f2003, Homework #8: Floating-Point Arithmetic (chapters 17-20, due Mon. Dec. 1)

Do the following problems from the textbook or defined below: 17.12, 17.C, 18.3, 19.15, 20.2

Problem 17.C (Logarithmic number systems): Consider an 8-bit unsigned number system in which the base-2 logarithm is represented with $k = 3$ whole and $l = 5$ fractional bits. **a.** What is the range of this number system? **b.** What is the maximum relative representation error for numbers within the above range? **c**. Represent the numbers $x = 7$ and $y = 11$ as accurately as possible in this number system.

Additional Practice Problems (chapters 21-24)

Because we have not done any homework on Chapters 21-24 of the textbook, the following problems are recommended for practice: 21.9a, 21.15, 22.5, 23.4, 24.5

Notes on homework solutions and other topics

A better solution to Problem 6.12 is based on the delay and cost recurrences $D(k) = D(\text{floor}(k/2)) + 2$ and $C(k) = C(\text{floor}(k/2)) + k - 1$ for a Brent-Kung network of general width k (not necessarily a power of 2).

In the solution handed out for Problem 11.12, the following corrections are needed for parts b and c. A couple of these are due to simple miscounting; one is due to the fact that in Wallace trees, HAs should not be used unless not doing so would increase the latency (number of levels).

Part b, line 5 -- 3 HAs; line 6 -- . . . 1 3 1 1 1 9-bit adder; line 7 -- 7 HAs + 9-bit adder

Part c, line 5 -- 4 FAs + 3 HAs; line 7 -- 21 FAs + 7 HAs

ECE 252B f2003, Possible Research Topics

In the following, IEEETC-7/2000 refers to the latest computer arithmetic special issue of *IEEE Trans. Computers* published in July 2000 and ARITH-2001 (2003) stands for the *Proc. 15th (16th) IEEE Symp. Computer Arithmetic* held in June 2001 (June 2003).

1. Design of parallel-prefix adders, IEEETC-7/2000 pp. 659-680, ARITH-2001 pp. 218-225 + 277-284.

2. Fast multiplication and powering, IEEETC-7/2000 pp. 681-701, ARITH-2001 pp. 23-39 + 73-79, ARITH-2003 pp. 204-211.

3. Floating-point normalization and rounding, IEEETC-7/2000 pp. 638-658, ARITH-2001 pp. 7-12 + 173-194, ARITH-2003 pp. 95-103.

4. Logarithmic arithmetic units, IEEETC-7/2000 pp. 702-726, ARITH-2001 pp. 229-245, ARITH-2003 pp. 253-261; Also [Cole00].

5. Function evaluation by table lookup, ARITH-2001 pp. 101-108 + 128-135, ARITH-2003 pp. 114-121

6. Arithmetic for cryptography, IEEETC-7/2000 pp. 740-758, ARITH-2001 pp. 59-65, ARITH-2003 pp. 174-202.

Midterm Exam Preparation

Textbook sections that are not required for the closed-book midterm exam: 3.5, 3.6, 4.4, 4.5, 4.6, 6.3, 7.2, 10.5 (some of these may be included in the final exam, which will be open-book)

Final Exam Preparation

The final exam will include Chapters 1-19 and 21-24, emphasizing the topics covered after the midterm. The following sections are excluded: 3.5 (pp. 45-48), 3.6, 4.4, 4.5, 4.6, 7.2, 10.5, 15.4, 15.6, 19.4, 21.4, 21.6, 22.6. All other sections are included, even if they were not discussed in class.

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ECE 252B: Fall Quarter 2002 offering

Course:	ECE 252B – Computer Arithmetic, University of California, Santa Barbara, Fall 2002, Enrollment Code 11700
Catalog entry:	**252B. Computer Arithmetic.** **(4) PARHAMI.** *Prerequisites: ECE 152A-B. Lecture, 4 hours.* Standard and unconventional number representations. Design of fast two-operand and multioperand adders. High-speed multiplication and division algorithms. Floating-point numbers, algorithms, and errors. Hardware algorithms for function evaluation. Pipelined, digit-serial and fault-tolerant arithmetic processors. (F)
Instructor:	Behrooz Parhami, Room 5155 Engineering I, Phone 805-893-3211, parhami at ece.ucsb.edu
Meetings:	MW 4:00-5:30 PM, Room 1425 Phelps Hall
Consultation:	Open office hours, held in Room 5155 Engineering I – M 11:00-12:30, W 12:30-2:00
Motivation:	Computer arithmetic is a subfield of digital computer organization. It deals with the hardware realization of arithmetic functions to support various computer architectures as well as with arithmetic algorithms for firmware/software implementation. A major thrust of digital computer arithmetic is the design of hardware algorithms and circuits to enhance the speed of various numeric operations. Thus much of what is presented in this course complements the *architectural* and *algorithmic* speedup techniques covered as part of the advanced computer architecture (ECE 254A/B/C) sequence.
Prerequisites:	Familiarity with logic design and switching theory as well as fundamentals of digital system design (ECE 152A and ECE 152B or equivalents).
References:	***Required textbook*** *(available at the bookstore)* Parhami, B., *Computer Arithmetic: Algorithms and Hardware Designs*, Oxford, 2000 (use the link to get information on the textbook and its errata). ***Other books, not required***

Swartzlander, *Computer Arithmetic*, vols. 1-2, IEEE Computer Soc. Press (QA76.6.C633).
Koren, *Computer Arithmetic Algorithms*, Prentice-Hall, 1993 (QA76.9.C62K67).
Omondi, *Computer Arithmetic Systems: . . .* , Prentice-Hall, 1994 (QA76.9.C62O46).
Ercegovac/Lang, *Division and Square Root: . . .* , Kluwer, 1994 (QA76.9.C62E73).
Oklobdzija, *High-Performance System Design*, IEEE Press, 1999 (TK7871.99.M44037).
Waser/Flynn, *Intro. to Arithmetic for Digital Systems Designers* (TK7895.A65W37.1982).
Knuth, *The Art of Computer Programming: Seminumerical Algor's* (QA76.6.K64 vol 2).
Hwang, *Computer Arithmetic: Principles, Architecture, and Design* (TK7888.3.H9).
Kulisch/Miranker, *Computer Arithmetic in Theory and Practice* (QA162.K84).
Research Sources
Proc. Symp. Computer Arithmetic (1969, 72, 75 78, and odd years since 1981).
IEEE Trans. Computers, particularly special issues on computer arithmetic (8/70, 6/73, 7/77, 4/83, 8/90, 8/92, 8/94, 7/98, 7/00).

Evaluation: Students will be evaluated based on the following three components with the given weights:

25% -- Homework (see the course calendar for general requirements and schedule).

25% -- Closed-book midterm exam (see the course calendar for date and coverage).

50% -- Open-book final exam (see the course calendar for date, time, and coverage)

Research: An optional research paper may be substituted for the final exam. A student interested in this option, will review a subfield of computer arithmetic or do original research on a selected topic. A list of research topics is provided below; however, please feel free to propose your own topic for approval. A publishable report earns an “A” for the course, regardless of homework and midterm grades. See the “deadlines” column in course calendar for schedule and due dates.

Calendar:

Course lectures, homework assignments, and exams have been scheduled as follows. This schedule will be strictly observed. Textbook chapters are specified for lectures and exams. It is a good idea to look over the specified material *before* each lecture if possible.

Day & Date	Chap	Lecture Topic	Deadlines and Notes
Mon. 9/30/02	1-2	Numbers and signed values	
Wed. 10/2/02	3-4	Redundant and RNS representations	HW#1 posted (chapters 1-4)
Mon. 10/7/02	5	Basic addition and counting	
Wed. 10/9/02	6	Carry-lookahead adders	HW#2 posted (5-6), HW #1 due
Mon. 10/14/02	7	Variations in fast adders	Research topic defined
Wed. 10/16/02	8	Multioperand addition	HW#3 posted (7-8), HW #2 due
Mon. 10/21/02	9	Basic multiplication schemes	
Wed. 10/23/02	10	High-radix multiplication	HW#4 posted (9-10), HW #3 due
Mon. 10/28/02	11	Tree and array multipliers	Preliminary references due
Wed. 10/30/02	12	Variations in multipliers (begins by 4:15)	HW #4 due
Mon. 11/4/02	1-10	Midterm exam: closed book (4-6 PM)	Note the extended time
Wed. 11/6/02		Instructor at conference: No lecture	HW#5 posted (11-12)
Mon. 11/11//02		Veterans' Day Holiday: No lecture	
Wed. 11/13/02	13	Basic division schemes	HW#6 posted (13-14), HW #5 due
Mon. 11/18/02	14	High-radix division	Paper title and references due
Wed. 11/20/02	15-16	Other division topics	HW#7 posted (15-16), HW #6 due
Mon. 11/25/02	17-18	Floating-point numbers & operations	Preliminary paper abstract due
Wed. 11/27/02	19-20	Errors, precision, & certifiability	HW#8 posted (17-20), HW #7 due
Mon. 12/2/02	21	Square-rooting methods	
Wed. 12/4/02	22	The CORDIC algorithms	HW #8 due
Mon. 12/9/02	23-24	Other topics in function evaluation	Complete paper due
Wed. 12/11/02	1-22	Final exam: open book (4-7 PM)	Phelps 1425 (our regular classroom)

Homework: General Requirements

Solutions should be brought to class on the due date and handed in before the lecture begins.
Late homework will not be accepted, so plan to start work on your assignments early.

Use a cover page that includes your name, course and assignment number for your solutions.
Staple the sheets and write your name on top of every sheet in case sheets are separated.
Although some cooperation is permitted, direct copying will have severe consequences.

Course Grade Stats

Item	Out of	Max	Mean	Median	Min	SD
HW #1	100	94	74	81	32	18
HW #2	100	98	75	84	30	26
HW #3	100	95	70	70	50	14
HW #4	100	96	83	86	42	17
HW #5	100	100	79	78	63	11
HW #6	100	100	87	87	72	8
HW #7	100	90	69	70	46	14
HW #8	100	90	77	80	51	13
Midterm	100	83	73	74	63	7
Final/Paper	100	90	72	69	57	13
Course*	100	87	75	73	61	10
Course	A (4.0)	A+	3.3	B	C+	0.66

* Course = 25%(HW total / 8) + 25%(Midterm) + 50%(Final or Paper)

ECE 252B f2002, Homework #1: Number Representation (chapters 1-4, due Wed. Oct. 9)

Do the following problems from the textbook: 1.4, 2.5, 2.7, 3.2, 4.3

ECE 252B f2002, Homework #2: Basic & Carry-Lookahead Addition (chapters 5-6, due Wed. Oct. 16)

Do the following problems from the textbook or defined below: 5.5e-h, 5.D, 6.3, 6.15, 6.H

Problem 5.D Full adder hardware realization Realize a full adder by means of a minimum number of 2-to-1 multiplexers and no other logic component (not even inverters).

Problem 6.H Designing fast comparators Given a fast carry network of any design, show how it can be used to build a fast comparator to determine if $x > y$, where x and y are unsigned binary integers.

ECE 252B f2002, Homework #3: Other Addition Topics (chapters 7-8, due Wed. Oct. 23)

Do the following problems from the textbook or defined below: 7.7, 7.18, 8.9, 8.14, 8.A

Problem 7.18 Correction In part c, fractional precision modes are 4 ´ (8-bit) or 2 ´ (16-bit).

Problem 8.A Generalized parallel counters **(a)** Show an implementation of a (5, 5; 4)-counter using (3; 2)-counters. **(b)** One level of (5, 5; 4)-counters can be used to reduce five operands to two. What is the maximum number of operands that can be reduced to two when two levels of (5, 5; 4)-counters are used? **(c)** Generalize the result of part b to a counter that reduces x columns of n dots to a $2x$-bit result.

ECE 252B f2002, Homework #4: Basic & High-Radix Multiplication (chapters 9-10, due Wed. Oct. 30)

Do the following problems from the textbook or defined below: 9.11, 9.14e-g, 9.D, 10.3, 10.A

Problem 9.14 Correction The last three parts should be labeled e, f, and g.

Problem 9.D Sequential 2's-complement multiplication **(a)** Represent $x = 3$, $y = –3$, and $z = 5$ as 4-bit, 2's-complement numbers. **(b)** Compute x ´ z to get the 8-bit product, using the sequential algorithm with right shifts. **(c)** Compute y ´ z to get the 8-bit product, using the sequential algorithm with left shifts.

Problem 10.A Radix-4 Booth's recoding Show that radix-4 Booth's recoding is equivalent to the following scheme: **(1)** Begin with a radix-4 operand employing the standard digit set [0, 3]. **(2)** Rewrite each 2 (3) digit as –2 (–1), with a radix-4 transfer of 1 to the next higher position. This results in an interim digit set [–2, 1]. **(3)** Add the transfers to the interim digits to obtain the recoded number with the digit set [–2, 2]. At the most significant end, ignore the outgoing transfer for a 2's-complement operand.

ECE 252B f2002, Homework #5: Other Multiplication Topics (chapters 11-12, due Wed. Nov. 13)

Do the following problems from the textbook or defined below: 11.1, 11.11a, 12.2, 12.12, 12.F

Problem 12.F Building larger AMMs A 6 ´ 2 AMM computes an 8-bit value $p = a$ ´ $x + b + y$, where a and b (x and y) are 6-bit (2-bit) unsigned numbers. Using dot notation, show how three such AMMs can be used to build a 6 ´ 6 AMM having two 6-bit multiplicative and two 6-bit additive inputs and producing a 12-bit result.

ECE 252B f2002, Homework #6: Basic & High-Radix Division (chapters 13-14, due Wed. Nov. 20)

Do the following problems from the textbook or defined below: 13.5bc, 13.12bc, 13.C, 14.1ab, 14.10ab

Problem 13.C Sequential nonrestoring division Apply the following modified form of nonrestoring division to the unsigned division .101/.110. In each iteration, the shifted partial remainder is compared to $\pm d$; if it is ³ d (< $-d$), $q_i = 1$ (¯1) is chosen; otherwise, q_i is set to 0. What do you think of this algorithm?

ECE 252B f2002, Homework #7: Other Division Topics (chapters 15-16, due Mon. Nov. 27)

Do the following problems from the textbook or defined below: 15.6, 15.11, 15.12, 16.3, 16.A

Problem 16.A Sequential versus convergence division Suppose multiplication and addition take 5 and 1 time units, respectively, while all support and control functions (counting, conditionals, register transfers, etc.) take negligible time due to overlapped processing. Be brief and state all your assumptions clearly. **(a)** Express the time needed for simple binary restoring division as a function of the word width k. **(b)** Express the time needed for division by repeated multiplications (without an initial table lookup or other speedup methods) as a function of k. **(c)** Compare the results of parts a and b. Comment on the speed/cost tradeoffs for different word widths.

ECE 252B f2002, Homework #8: Floating-Point Arithmetic (chapters 17-20, due Mon. Dec. 4)

Do the following problems from the textbook or defined below: 17.10, 17.15, 18.6bc, 19.C, 20.2

Problem 19.C Backward error analysis Suppose we compute x^4 by squaring x and again squaring the result. Each squaring operation is done via a floating-point multiplication. Show that the computed result is $z = [(1 + r)x]^4$ and establish a bound on the magnitude of r.

Notes on homework solutions and other topics

Practice problems from Chapters 21-22 (solutions will be handed out by 2002/12/9): 21.9a, 21.11, 22.1, 22.12

ECE 252B f2002, Possible Research Topics

The following is from the previous offering of the course in fall 2001. It will be updated early in the fall quarter to reflect new topics and references.

In the following, IEEETC-7/2000 refers to the latest computer arithmetic special issue of *IEEE Trans. Computers* published in July 2000 and ARITH-2001 stands for the *Proc. 15th IEEE Symp. Computer Arithmetic* held in June 2001.

1. Design of parallel-prefix adders, IEEETC-7/2000 pp. 659-680, ARITH-2001 pp. 218-225 + 277-284.

2. Fast multiplication and squaring, IEEETC-7/2000 pp. 681-701, ARITH-2001 pp. 23-39 + 73-79.

3. Floating-point normalization and rounding, IEEETC-7/2000 pp. 638-658, ARITH-2001 pp. 7-12 + 173-194.

4. Logarithmic arithmetic units, IEEETC-7/2000 pp. 702-726, ARITH-2001 pp. 229-245. Also [Cole00].

5. Function evaluation by table lookup, ARITH-2001 pp. 101-108 + 128-135

6. Arithmetic for cryptography, IEEETC-7/2000 pp. 740-758, ARITH-2001 pp. 59-65.

Midterm Exam Preparation

Textbook sections that are not required for the closed-book midterm exam: 3.5 (pp. 45-48), 3.6, 4.4, 4.5, 4.6, 6.3, 7.2 (some of these may be included in the final exam, which will be open-book)

Final Exam Preparation

The final exam will include Chapters 1-19 and 21-22, emphasizing the topics covered after the midterm. The following sections are excluded: 3.5 (pp. 45-48), 3.6, 4.4, 4.5, 4.6, 7.2, 15.4, 15.6, 19.4, 21.4, 21.6, 22.6. All other sections are included, even if they were not discussed in class.

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ECE 252B: Fall Quarter 2001 offering

Course:	ECE 252B – Computer Arithmetic, University of California, Santa Barbara, Fall 2001, Enrollment Code 58859
Instructor:	Behrooz Parhami, Room 5155 Engineering I, Phone 805-893-3211, parhami at ece.ucsb.edu
Meetings:	MW 4:00-5:30 PM, Room 1174 HSSB
Consultation:	Open office hours, held in Room 5155 Engineering I – M 9-10, W 1-2, R 3-4
Motivation:	Computer arithmetic is a subfield of digital computer organization. It deals with the hardware realization of arithmetic functions to support various computer architectures as well as with arithmetic algorithms for firmware/software implementation. A major thrust of digital computer arithmetic is the design of hardware algorithms and circuits to enhance the speed of various numeric operations. Thus much of what is presented in this course complements the *architectural* and *algorithmic* speedup techniques covered as part of the advanced computer architecture (ECE 254A/B/C) sequence.
Prerequisites:	Familiarity with logic design and switching theory as well as fundamentals of digital system design (ECE 152A and ECE 152B or equivalents).
References:	***Required textbook*** *(available at the bookstore)* Parhami, B., *Computer Arithmetic: Algorithms and Hardware Designs*, Oxford, 2000 (use the link to get information on the textbook and its errata). ***Recommended book*** *(available at the bookstore, also to be put on 2-hour reserve)* Flynn/Oberman, *Advanced Computer Arithmetic Design*, Wiley, 2001 (TK7895.A65F59). ***Other books, not required*** *(* indicates that the book is to be put on 2-hour reserve)* *von zur Gathen/Gerhard, *Modern Computer Algebra*, Cambridge, 1999. *Swartzlander, *Computer Arithmetic*, vols. 1-2, IEEE Computer Soc. Press (QA76.6.C633). *Koren, *Computer Arithmetic Algorithms*, Prentice-Hall, 1993 (QA76.9.C62K67). *Omondi, *Computer Arithmetic Systems: . . .* , Prentice-Hall, 1994 (QA76.9.C62O46). *Ercegovac/Lang, *Division and Square Root: . . .* , Kluwer, 1994 (QA76.9.C62E73). *Oklobdzija, *High-Performance System Design*, IEEE Press, 1999 (TK7871.99.M44037). Waser/Flynn, *Intro. to Arithmetic for Digital Systems Designers* (TK7895.A65W37.1982). Knuth, *The Art of Computer Programming: Seminumerical Algor's* (QA76.6.K64 vol 2). Hwang, *Computer Arithmetic: Principles, Architecture, and Design* (TK7888.3.H9). Kulisch/Miranker, *Computer Arithmetic in Theory and Practice* (QA162.K84). ***Research Sources*** *Proc. Symp. Computer Arithmetic* (1969, 72, 75 78, and odd years since 1981). *IEEE Trans. Computers*, particularly special issues on computer arithmetic (8/70, 6/73, 7/77, 4/83, 8/90, 8/92, 8/94, 7/98, 7/00).
Evaluation:	Students will be evaluated based on the following three components with the given weights: 25% -- Homework (see the course calendar for general requirements and schedule). 25% -- Closed-book midterm exam (see the course calendar for date and coverage). 50% -- Open-book final exam (see the course calendar for date, time, and coverage)
Research:	An optional research paper may be substituted for the final exam. A student interested in this option, will review a subfield of computer arithmetic or do original research on a selected topic. A list of research topics is provided below; however, please feel free to propose your own topic for approval. A publishable report earns an "A" for the course, regardless of homework and midterm grades. See the "deadlines" column in course calendar for schedule and due dates.

Calendar:

Course lectures, homework assignments, and exams have been scheduled as follows. This schedule will be strictly observed. Textbook chapters are specified for lectures and exams. It is a good idea to look over the specified material *before* each lecture if possible.

Day & Date	Chap	Lecture Topic	Deadlines and Notes
Mon. 9/24/01	1-2	Numbers and signed values	
Wed. 9/26/01	3-4	Redundant and RNS representations	HW #1 posted (chapters 1-4)
Mon. 10/1/01	5	Basic addition and counting	
Wed. 10/3/01	6	Carry-lookahead adders	HW #2 posted (5-6), HW #1 due
Mon. 10/8/01	7	Variations in fast adders	Research topic defined
Wed. 10/10/01	8	Multioperand addition	HW #3 posted (7-8), HW #2 due
Mon. 10/15/01	9	Basic multiplication schemes	
Wed. 10/17/01	10	High-radix multiplication	HW#4 posted (9-10), HW #3 due
Mon. 10/22/01	11	Tree and array multipliers	Preliminary references due
Wed. 10/24/01	12	Variations in multipliers	HW #4 due
Mon. 10/29/01	1-10	Midterm exam: closed book (4-6 PM)	HW #5 posted (11-12)
Wed. 10/31/01	13	Basic division schemes	

Mon. 11/5//01	14	High-radix division	HW #6 posted (13-14), HW #5 due
Wed. 11/7/01		Instructor at conference: No lecture	
Mon. 11/12/01		Veterans' Day Holiday: No lecture	Paper title and references due
Wed. 11/14/01	15	Variations in dividers	HW #6 due
Mon. 11/19/01	16	Division by convergence	HW #7 posted (15-16)
Wed. 11/21/01	17-18	Floating-point numbers & operations	Preliminary paper abstract due
Mon. 11/26/01	19-20	Errors, precision, & certifiability	HW #8 posted (17-20), HW #7 due
Wed. 11/28/01	21	Square-rooting methods	
Mon. 12/3/01	22	The CORDIC algorithms	HW #8 due
Wed. 12/5/01	23-24	Other topics in function evaluation	
Fri. 12/7/01	1-22	Final exam: open book (9AM-12PM)	Complete paper due
		Final to be held in Girvetz 1106	

Homework: General Requirements

Solutions should be brought to class on the due date and handed in before the lecture begins.
Late homework will not be accepted, so plan to start work on your assignments early.
Use a cover page that includes your name, course and assignment number for your solutions.
Staple the sheets and write your name on top of every sheet in case sheets are separated.
Although some cooperation is permitted, direct copying will have severe consequences.

ECE 252B w2001, Homework #1: Number Representation (chapters 1-4, due Wed. Oct. 3)

Do the following problems from the textbook: 1.1, 1.6, 2.3, 2.9, 3.7, 4.11

ECE 252B w2001, Homework #2: Basic & Carry-Lookahead Addition (chapters 5-6, due Wed. Oct. 10)

Do the following problems from the textbook: 5.3, 5.6abc, 5.7, 6.13

Additional problem 6.C

Consider a 16-bit carry-lookahead adder with 4-bit blocks. Assume that block p and g signals are produced after 3 gate delays and that each block uses ripple-carry internally. The design uses a lookahead carry generator with 2 gate delays. Carry ripples through each stage in 2 gate delays and sum bits are computed in two gate delays once all the internal carries are known. State your assumptions whenever the information provided is not sufficient to answer the question.

a. Compute the total addition time, in terms of gate delays, for this 16-bit adder.

b. We gradually increase the adder width to 17, 18, 19, . . . bits using 4 ripple-carry groups of equal or approximately equal widths, while keeping the block p and g delay constant. At what word width k would it be possible to increase the adder speed by using an additional level of lookahead?

Bonus (optional) problem for extra credit: Problem 6.4 in the textbook.

ECE 252B w2001, Homework #3: Other Addition Topics (chapters 7-8, due Wed. Oct. 17)

Do the following problems from the textbook: 7.13, 7.18, 8.12, 8.13

Additional background and correction for Problem 7.18: Fractional precision is the opposite of multiple precision. It is used when several low-precision numbers (such as those found in multimedia data) are packed into a single word and processed in parallel. Adding two such words, for example, has the effect of several independent shorter additions. For this reason, the term "subword parallelism" has also been used to describe this approach. Subword arithmetic is often performed with saturation (see Problem 7.F below). In part c of the problem, "(4 x 8)-bit or (2 x 16)-bit" should be "4 x (8-bit) or 2 x (16-bit)".

Additional Problem 7.F

In certain applications, when the result of an arithmetic operation exceeds the maximum allowed value, it would be inappropriate to generate the result modulo a power of 2. For example, in media processing, we do not want addition of 1 to a black pixel coded as FF in hexadecimal to turn it into a white pixel 00. Discuss how two-operand addition can be performed with saturation so that whenever the result exceeds $2^k - 1$, the maximum value $2^k - 1$ is produced at output.

ECE 252B w2001, Homework #4: Basic & High-Radix Multiplication (chapters 9-10, due Wed. Oct. 24)

Do the following problems from the textbook: 9.4d, 9.10, 9.14abc, 10.1, 10.15

ECE 252B w2001, Homework #5: Other Multiplication Topics (chapters 11-12, due Wed. Nov. 5)

Do the following problems from the textbook: 11.3, 11.9, 11.17, 12.5a, 12.18

ECE 252B w2001, Homework #6: Basic & High-Radix Division (chapters 13-14, due Wed. Nov. 14)

Do the following problems from the textbook: 13.2, 13.9, 13.12d, 14.4cd (see below), 14.14

Correction to Problem 14.4: In all four parts, change the magnitude of the divisor *d* to .1010

ECE 252B w2001, Homework #7: Other Division Topics (chapters 15-16, due Mon. Nov. 26)

Do the following problems from the textbook: 15.3, 15.5, 16.6, 16.12

Additional Problem 15.B

Argue that the use of the quotient digit set [–6, 6] is preferable to the minimal digit set [–4, 4] in a radix-8 divider that forms the multiple 3*d* by inputting the two values 2*d* and *d* into a four-input carry-save adder tree that also receives the carry-save partial remainder as inputs.

ECE 252B w2001, Homework #8: Floating-Point Arithmetic (chapters 17-20, due Mon. Dec. 3)

Do the following problems from the textbook: 17.4, 17.8, 18.5, 18.18, 19.5

Bonus problem for extra credit: 20.4

ECE 252B f2001, Possible Research Topics

In the following, IEEETC-7/2000 refers to the latest computer arithmetic special issue of *IEEE Trans. Computers* published in July 2000 and ARITH-2001 stands for the *Proc. 15th IEEE Symp. Computer Arithmetic* held in June 2001.

1. Design of parallel-prefix adders, IEEETC-7/2000 pp. 659-680, ARITH-2001 pp. 218-225 + 277-284.

2. Fast multiplication and squaring, IEEETC-7/2000 pp. 681-701, ARITH-2001 pp. 23-39 + 73-79.

3. Floating-point normalization and rounding, IEEETC-7/2000 pp. 638-658, ARITH-2001 pp. 7-12 + 173-194.

4. Logarithmic arithmetic units, IEEETC-7/2000 pp. 702-726, ARITH-2001 pp. 229-245. Also [Cole00].

5. Function evaluation by table lookup, ARITH-2001 pp. 101-108 + 128-135

6. Arithmetic for cryptography, IEEETC-7/2000 pp. 740-758, ARITH-2001 pp. 59-65.

Final Exam Preparation

The final exam will include Chapters 1-19 and 21-22, emphasizing the topics covered after the midterm. The following sections are excluded: 4.4, 4.5, 4.6, 19.4, 22.6. All other sections are included, even if they were not discussed in class.

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ECE 252B: Winter Quarter 2001 offering

Course:	ECE 252B – Computer Arithmetic, University of California, Santa Barbara, Winter 2001, Enrollment Code 11270
Instructor:	Behrooz Parhami, Room 5155 Engineering I, Phone 805-893-3211, parhami at ece.ucsb.edu
Meetings:	MW (plus review sessions on F 2/9 and 3/16) 8:30-9:50 AM, Room 1160 Phelps Hall
Consultation:	Open office hours, held in Room 5155 Engineering I – M 11-12, W 1-2, R 3-4
Motivation:	Computer arithmetic is a subfield of digital computer organization. It deals with the hardware realization of arithmetic functions to support various computer architectures as well as with arithmetic algorithms for firmware/software implementation. A major thrust of digital computer arithmetic is the design of hardware algorithms and circuits to enhance the speed of various numeric operations. Thus much of what is presented in this course complements the *architectural* and *algorithmic* speedup techniques covered as part of the advanced computer architecture (ECE 254A/B/C) sequence.
Prerequisites:	Familiarity with logic design and switching theory as well as fundamentals of digital system design (ECE 152A and ECE 152B or equivalents).
References:	***Required textbook*** Parhami, B., *Computer Arithmetic: Algorithms and Hardware Designs*, Oxford, 2000 (use the link to get information on the textbook and its errata). ***Other books*** *(not required)* Koren, *Computer Arithmetic Algorithms*, Prentice-Hall, 1993 (QA76.9.C62K67).

Omondi, *Computer Arithmetic Systems: . . . ,* Prentice-Hall, 1994 (QA76.9.C62O46).
Waser/Flynn, *Intro. to Arithmetic for Digital Systems Designers* (TK7895.A65W37.1982).
Knuth, *The Art of Computer Programming: Seminumerical Algor's* (QA76.6.K64 vol 2).
Hwang, *Computer Arithmetic: Principles, Architecture, and Design* (TK7888.3.H9).
Swartzlander, *Computer Arithmetic*, vols. 1-2, IEEE Computer Soc. Press (QA76.6.C633).
Kulisch/Miranker, *Computer Arithmetic in Theory and Practice* (QA162.K84).
Ercegovac/Lang, *Division and Square Root: . . . ,* Kluwer, 1994 (QA76.9.C62E73).
Research Sources
Proc. Symp. Computer Arithmetic (1969, 72, 75 78, and odd years since 1981).
IEEE Trans. Computers, particularly special issues on computer arithmetic (8/70, 6/73, 7/77, 4/83, 8/90, 8/92, 8/94, 7/98, 7/00).

Evaluation: Students will be evaluated based on the following three components with the given weights:

25% -- Homework (see the course calendar for general requirements and schedule).

25% -- Closed-book midterm exam (see the course calendar for date and coverage).

50% -- Open-book final exam (see the course calendar for date, time, and coverage)

Research: An optional research paper may be substituted for the final exam. A student interested in this option, will review a subfield of computer arithmetic or do original research on a selected topic. A list of research topics for winter 2001 is provided below; however, please feel free to propose your own topic for approval. A publishable report earns an "A" for the course, regardless of homework and midterm grades. See the "deadlines" column in course calendar for schedule and due dates.

Calendar:

Course lectures, homework assignments, and exams have been scheduled as follows. This schedule will be strictly observed. Textbook chapters are specified for lectures and exams. It is a good idea to look over the covered chapter(s) *before* each lecture if possible.

Day & Date	Chap	Lecture Topic	Deadlines and Notes
Mon. 1/8/01	1-2	Numbers and signed values	
Wed. 1/10/01	3-4	Redundant and RNS representations	
Mon. 1/15/01		Dr. King's Birthday: No lecture	HW#1 posted (chapters 1-4)
Wed. 1/17/01	5	Basic addition and counting	
Mon. 1/22/01	6	Carry-lookahead adders	HW#2 posted (5-6), HW #1 due
Wed. 1/24/01	7	Variations in fast adders	Research topic defined
Mon. 1/29/01	8	Multioperand addition	HW#3 posted (7-8), HW #2 due
Wed. 1/31/01	9	Basic multiplication schemes	
Mon. 2/5/01	10	High-radix multiplication	HW#4 posted (9-11), HW #3 due
Wed. 2/7/01	11	Tree and array multipliers	Preliminary references due
Fri. 2/9/01		Review (8:30-9:50 AM, 1160 Phelps)	
Mon. 2/12/01	12	Variations in multipliers	HW #4 due
Wed. 2/14/01	1-12	Midterm exam: closed book (8-10 AM)	
Mon. 2/19/01		President's Day: No lecture	HW#5 posted (12-13)
Wed. 2/21/01	13	Basic division schemes	
Mon. 2/26/01	14	High-radix division	HW#6 posted (14-16), HW #5 due
Wed. 2/28/01	15-16	Other division topics	Paper title and references due
Mon. 3/5/01	17	Floating-point numbers	HW#7 posted (17-19), HW #6 due
Wed. 3/7/01	18-19	Floating-point operations and errors	Preliminary paper abstract due
Mon. 3/12/01	21	Square-rooting methods	HW #7 due
Wed. 3/14/01	22	The CORDIC algorithms	
Fri. 3/16/01		Review (8:30-9:50 AM, 1160 Phelps)	
Mon. 3/19/01	1-22	Final exam*: open book (8:30-11 AM)	Complete paper due by 12:00 noon
		*In 2162 Engr I, not our regular classroom	

Homework: General Requirements

Solutions should be brought to class on the due date and handed in before the lecture begins.
Late homework will not be accepted, so plan to start work on your assignments early.
Use a cover page that includes your name, course and assignment number for your solutions.

Staple the sheets and write your name on top of every sheet in case sheets are separated.
Although some cooperation is permitted, direct copying will have severe consequences.

ECE 252B w2001, Homework #1: Number Representation (chapters 1-4, due Mon. Jan. 22)

Do the following five problems from the textbook:

1.2 (p. 15), 1.9 (p. 16), 2.10 (p. 32), 3.6 (p. 51), 4.5 (p. 70)

Hint for Problem 1.9: Compute the weights associated with the first 16 positions of a number.

Bonus problem 1.D on number representation (for extra credit): We want to build an abacus for use with the Roman numerals system. There are to be seven positions labeled, from left to right, M, D, C, L, X, V, and I. Each position is to have positive (black) and negative (red) beads to allow representations such as MCDXXIV. What are the minimum required numbers of the two types of beads in each position, given that all unsigned integers up to 1500 are to be representable?

ECE 252B w2001, Homework #2: Basic & Carry-Lookahead Addition (chapters 5-6, due Mon. Jan. 29)

Do the following five problems from the textbook:

5.5 parts j, l, m only (p. 88), 5.8 (p. 88), 5.9 (89), 6.1 (p. 104), 6.14 (p. 106)

ECE 252B w2001, Homework #3: Other Addition Topics (chapters 7-8, due Mon. Feb. 5)

Do the following four problems from the textbook and part a of problem 7.E given below:

7.4 (p. 121), 7.16 (p. 123), 8.6 (p. 138), 8.10 (p. 139)

Problem 7.E on self-timed carry-skip addition: (a) Apply the carry-skip idea to the self-timed adder of Fig. 5.9, illustrating the result by drawing a block diagram similar to that in Fig. 7.1 for a 16-bit carry-completion adder made of four 4-bit skip blocks. (b) [optional bonus part, for extra credit] Analyze the expected carry completion time in the adder of part a, when blocks of fixed width *b* are used to build a *k*-bit self-timed adder, and discuss the practicality of this idea.

Note (correction to the solution to Problem 3.6, part c): It was brought to my attention that the proposed solution does not work when the number has leading 0s. In this case, the leftmost borrow of 1 will not be absorbed, leaving a number with an invalid digit -1. There seems to be no way around this problem. A -1 digit must create a borrow which must be propagated over a sequence of 0 digits. If 0s are not transformed to positive values, the borrow cannot be absorbed. Because the length of the sequence of 0 digits is unbounded, no local transformation (carry-free process) can decide if the 0s are part of a sequence of leading 0s. If you feel that you made this point but did not receive proper credit, please see me.

Note (correction to the solution of Problem 6.1): Besides the misspelled "performing" in part a, the term "pi_i" (or b_i) in the expression for d_i in part b should be complemented.

ECE 252B w2001, Homework #4: Multiplication (chapters 9-11, due Mon. Feb. 12)

Do the following six problems from the textbook:

9.1 (p. 153), 9.4b (p. 154), 9.14a (p. 155), 10.1 (p. 169), 10.3 (p. 169), 10.15 (p. 171)

ECE 252B w2001, Homework #5: Multiplication & Division (chapters 12-13, due Mon. Feb. 26)

Do the following three problems from the textbook, plus Problem 12.A given below:

12.4 (p. 205), 13.10 (p. 225), 13.13 (p. 226)

Problem 12.A on synthesis of multipliers:

a. We want to synthesize a 12-by-4 parallel multiplier using single-bit and 4-bit binary adders as the only building blocks. Using dot notation, specify an efficient design that minimizes the cost, assuming unit cost for 1-bit adders and 6 units for 4-bit adders.

b. Repeat part a, this time using only (4, 4; 4)-counters and 4-bit adders.

c. Repeat part a, this time using 4-by-4 multipliers and 4-bit adders as the only building blocks.

ECE 252B w2001, Homework #6: Other Division Topics (chapters 14-16, due Mon. Mar. 5)

Do the following three problems from the textbook, plus Problem 16.B given below:

14.4a,b (p. 242), 14.8 (p. 243), 15.4 (p. 257)

Optional problem (for extra credit): 16.10 (p. 274)

Problem 16.B on iterative reciprocation:

a. Compute the reciprocal of d = (.1100 0000)_two using x^(i+1) = x^(i) (2 - x^(i) d) and arithmetic with 12 bits after the radix point throughout.

b. Repeat part a, using the expansion 1/d = 1/(1 - y) = (1 + y)(1 + y^2)(1 + y^4)..., where y = 1 - d, instead of the Newton-Raphson iteration. Each term 1 + y^(2^(i+1)) is computed by squaring y^(2^i) and adding 1.

c. Compare the results of parts a and b and discuss.

ECE 252B w2001, Homework #7: Floating-Point Arithmetic (chapters 17-19, due Mon. Mar. 12)

[Note on HW#6: The statement of Problem 16.10 seems to be in error and I have been unable to ascertain what the correct version is. If you are able to find the correct version of the suggested convergence formula, then do the problem. Otherwise, you can do Problem 16.11 instead, which is quite similar and serves the same purpose as the one originally assigned.]

Do the following four problems from the textbook:

17.7 (p. 294), 17.14 (p. 295), 18.9 (p. 309), 19.9 (p. 325)

Optional problem (for extra credit): 19.10 (p. 326)

Notes on MRRE for logarithmic representation

Consider two consecutive logarithms L and L + ulp and a number x that falls between 2^L and 2^(L + ulp).

Relative error for rounding down = (x - 2^L)/x

Relative error for rounding up = (2^(L + ulp) - x)/x

The worst-case occurs when the relative error is the same for rounding down or up. Thus, the worst-case x satisfies:

(x - 2^L)/x = (2^(L + ulp) - x)/x

From the above, we get x = (2^(L + ulp) + 2^L)/2. Plug this into either relative error equation to get:

MRRE = (2^ulp - 1)/(2^ulp + 1)

Because 2^ulp is very close to 1, MRRE is approximately (2^ulp - 1)/2

ECE 252B w2001, Possible Research Topics

The ECE 252B research topics for winter 2001 are based on the latest computer arithmetic special issue of *IEEE Trans. Computers* published in July 2000. The special issue contains six pairs of papers in the following areas. Each pair can be chosen as a research subject area. The research report will be based on a detailed study of the chosen pair of papers, plus additional references that the student will identify in order to prepare a complete and informative paper. The six subject areas are:

1. Function evaluation = reciprocation & square-rooting, pp. 628-637 + CORDIC, pp. 727-739.

2. Rounding, pp. 638-658.

3. Addition, pp. 659-680.

4. Multiplication, pp. 681-701.

5. Logarithmic processors, pp. 702-726.

6. Cryptography, pp. 740-758.

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ECE 252B: Winter Quarter 2000 offering

Course:	ECE 252B – Computer Arithmetic, University of California, Santa Barbara, Winter 2000, Enrollment Code 10686
Instructor:	Behrooz Parhami, Room 5155 Engineering I, Phone 805-893-3211, parhami at ece.ucsb.edu

Meetings: MW 8:30-10:00 AM, Room 1437 Phelps Hall

Consultation: Open office hours (held in Room 5155 Engineering I) – M 10:00-11:00, T 11:30-12:30, W 1:00-2:00

Motivation: Computer arithmetic is a subfield of digital computer organization. It deals with the hardware realization of arithmetic functions to support various computer architectures as well as with arithmetic algorithms for firmware/software implementation. A major thrust of digital computer arithmetic is the design of hardware algorithms and circuits to enhance the speed of various numeric operations. Thus much of what is presented in this course complements the *architectural* and *algorithmic* speedup techniques covered as part of the advanced computer architecture (ECE 254A/B/C) sequence.

Prerequisites: Familiarity with logic design and switching theory as well as fundamentals of digital system design (ECE 152A and ECE 152B or equivalents).

References: *Text*–Parhami, B., *Computer Arithmetic: Algorithms and Hardware Designs*, Oxford, 2000.
Other books – Koren, *Computer Arithmetic Algorithms*, Prentice-Hall, 1993.
Omondi, *Computer Arithmetic Systems: . . .* , Prentice-Hall, 1994 (QA76.9.C62O46).
Waser/Flynn, *Intro. to Arithmetic for Digital Systems Designers* (TK7895.A65W37.1982).
Knuth, *The Art of Computer Programming: Seminumerical Algorithms* (QA76.6.K64 vol 2).
Hwang, *Computer Arithmetic: Principles, Architecture, and Design* (TK7888.3.H9).
Swartzlander, *Computer Arithmetic*, vols. 1-2, IEEE Computer Soc. Press (QA76.6.C633).
Kulisch/Miranker, *Computer Arithmetic in Theory and Practice* (QA162.K84).
Ercegovac/Lang, *Division and Square Root: Digit-Recurrence . . .* , Kluwer, 1994.
Other Sources – Proc. Int'l Symp. on Computer Arithmetic (1969, 72, 75 78, and odd years since 1981) and *IEEE Trans. Computers*, particularly special issues on computer arithmetic (8/70, 6/73, 7/77, 4/83, 8/90, 8/92, 8/94, 7/98)

Evaluation: Students will be evaluated based on the following three components with the given weights:

25% -- Homework (see the "deadlines" column in course calendar for schedule).

25% -- Closed-book midterm exam (see the course calendar for date and coverage).

50% -- Open-book final exam (see the course calendar for date, time, and coverage)

Research: An optional research paper may be substituted for the final. A student interested in this option, will review a subfield of computer arithmetic or do original research on a selected topic. A publishable report earns an "A" for the course, regardless of homework and midterm grades. See the "deadlines" column in course calendar for schedule and due dates.

Calendar:

Course lectures, homework assignments, and exams have been scheduled as follows. This schedule will be strictly observed. Textbook chapters are specified for lectures and exams. It is a good idea to look over the covered chapter(s) *before* each lecture if possible.

Day and Date	Chap	Lecture Topic	Deadlines
Mon. 1/10/00	1-2	Numbers and signed values	
Wed. 1/12/00	3-4	Redundant and RNS representations	HW#1 given (1-4)
Mon. 1/17/00		HOLIDAY: No lecture	
Wed. 1/19/00	5-6	Basic and carry-lookahead addition	HW #2 given (5-6), #1 due
Mon. 1/24/00	7	Variations in fast adders	Research topics defined
Wed. 1/26/00	8	Multioperand addition	HW #3 given (7-8), #2 due
Mon. 1/31/00	9-10	Basic and high-radix multiplication	
Wed. 2/2/00	11	Tree and array multipliers	HW #4 given (9-12), #3 due
Mon. 2/7/00	12	Variations in multipliers	Preliminary references due
Wed. 2/9/00	13	Basic division schemes	HW #4 due
Mon. 2/14/00	1-12	MIDTERM EXAM: closed book	
Wed. 2/16/00	14	High-radix division	HW #5 given (13-14)
Mon. 2/21/00		HOLIDAY: No lecture	
Wed. 2/23/00	15-16	Other division topics	HW #6 given (15-16), #5 due
Mon. 2/28/00	17-18	Floating-point arithmetic	Paper title and references due
Wed. 3/1/00	19-20	Computation errors and precision	HW #7 given (17-20), #6 due
Mon. 3/6/00	21	Square-rooting methods	Preliminary paper abstract due
Wed. 3/8/00	22	The CORDIC algorithms	HW #8 given (21-24), #7 due
Mon. 3/13/00	23-24	Function evaluation	
Wed. 3/15/00	25-28	Implementation topics	HW #8 due
Fri. 3/24/00	1-24	FINAL EXAM: open book	Complete paper due

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