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Appendix: Past, Present, and Future
About This Presentation

This presentation is intended to support the use of the textbook *Dependable Computing: A Multilevel Approach* (traditional print or on-line open publication, TBD). It is updated regularly by the author as part of his teaching of the graduate course ECE 257A, Fault-Tolerant Computing, at Univ. of California, Santa Barbara. Instructors can use these slides freely in classroom teaching or for other educational purposes. Unauthorized uses, including distribution for profit, are strictly prohibited. © Behrooz Parhami

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How the Cover Image Relates to Our Course

Dependability as weakest-link attribute:
Under stress, the weakest link will break, even if all other links are superstrong

- Improve the least reliable part first

Safety factor (use of redundancy):
Provide more resources than needed for the minimum acceptable functionality

Additional resources not helpful if:
- failures are not independent
- Some critical component fails
About the Name of This Course

Fault-tolerant computing: a discipline that began in the late 1960s – 1st Fault-Tolerant Computing Symposium (FTCS) was held in 1971

In the early 1980s, the name “dependable computing” was proposed for the field to account for the fact that tolerating faults is but one approach to ensuring reliable computation. The terms “fault tolerance” and “fault-tolerant” were so firmly established, however, that people started to use “dependable and fault-tolerant computing.”

In 2000, the premier conference of the field was merged with another and renamed “Int’l Conf. on Dependable Systems and Networks” (DSN)

In 2004, IEEE began the publication of *IEEE Trans. On Dependable and Secure Systems* (inclusion of the term “secure” is for emphasis, because security was already accepted as an aspect of dependability)
Why This Course Shouldn’t Be Needed

In an ideal world, methods for dealing with faults, errors, and other impairments in hardware and software would be covered within every computer engineering course that has a design component.

**Analogy:** We do not teach structural engineers about building bridges in one course and about bridge safety and structural integrity during high winds or earthquakes in another (optional) course.

- **Logic Design:** fault testing, self-checking
- **Parallel Comp.:** reliable commun., reconfiguration
- **Programming:** bounds checking, checkpointing

Fault-Tolerant Computing
Brief History of Dependable Computing

1940s: ENIAC, with 17.5K vacuum tubes and 1000s of other electrical elements, failed once every 2 days (avg. down time = minutes)

1950s: Early ideas by von Neumann (multichannel, with voting) and Moore-Shannon (“crummy” relays)

1960s: NASA and military agencies supported research for long-life space missions and battlefield computing

1970s: The field developed quickly (international conference, many research projects and groups, experimental systems)

1980s: The field matured (textbooks, theoretical developments, use of ECCs in solid-state memories, RAID concept), but also suffered some loss of focus and interest because of the extreme reliability of integrated circuits

1990s: Increased complexity at chip and system levels made verification, testing, and testability prime study topics

2000s: Resurgence of interest owing to less reliable fabrication at ultrahigh densities and “crummy” nanoelectronic components
Dependable Computing in the 2010s

There are still ambitious projects; space and elsewhere
Harsh environments (vibration, pressure, temperatures)
External influences (radiation, micrometeoroids)
Need for autonomy (commun. delays, unmanned probes)

The need is expanding
More complex systems (e.g., system-on-chip)
Critical applications (medicine, transportation, finance)
Expanding pool of unsophisticated users
Continued rise in maintenance costs
Digital-only data (needs more rigorous backup)

The emphasis is shifting
Mostly COTS-based solutions
Integrated hardware/software systems
Entire units replaced (system-level diagnosis)
Pretest: Failures and Probabilities

This test will not be graded or even collected, so answer the test questions truthfully and to the best of your ability/knowledge.

**Question 1:** Name a disaster that was caused by computer hardware or software failure. How do you define “disaster” and “failure”?

**Question 2:** Which of these patterns is more random?

**Question 3:** Which do you think is more likely: the event that everyone in this class was born in the first half of the year or the event that at least two people were born on the same day of the year?

**Question 4:** In a game show, there is a prize behind one of 3 doors with equal probabilities. You pick Door A. The host opens Door B to reveal that there is no prize behind it. The host then gives you a chance to switch to Door C. Is it better to switch or to stick to your choice?
Question 5: Does this photo depict a mishap due to design flaw, implementation bug, procedural inadequacies, or human error?
Pretest (Continued): Reliability and Risk

Question 6: Name an emergency backup system (something not normally used unless another system fails) that is quite commonplace.

Question 7: Which is more reliable: Plane X or Plane Y that carries four times as many passengers as Plane X and is twice as likely to crash?

Question 8: Which is more reliable: a 4-wheel vehicle with one spare tire or an 18-wheeler with 2 spare tires?

Question 9: Which surgeon would you prefer for an operation that you must undergo: Surgeon A, who has performed some 500 operations of the same type, with 5 of his patients perishing during or immediately after surgery, or Surgeon B, who has a perfect record in 25 operations?

Question 10: Which is more probable at your home or office: a power failure or an Internet outage? Which is likely to last longer?

If you had trouble with 3 or more questions, you really need this course!
August 1, 2007 – Interstate 35W
Bridge 9340 over the Mississippi, in Minneapolis
(40-year old bridge was judged structurally deficient in 1990)
History of Bridge 9340 in Minneapolis

1967: Opens to traffic

1990: Dept. of Transportation classifies bridge as “structurally deficient”

1993: Inspection frequency doubled to yearly

1999: Deck and railings fitted with de-icing system

2001: U. Minn. engineers deem bridge struct. deficient

2004-07: Fatigue potential and remedies studied

2007: Inspection plan chosen over reinforcements

Summer 2007: $2.4M of repairs/maintenance on deck, lights, joints

Aug. 1, 2007: Collapses at 6:05 PM, killing 7

Sep. 18, 2008: Replacement bridge opens
What Do We Learn from Bridges that Collapse?

Opening day of the Tacoma Narrows Bridge, July 1, 1940

Nov. 7, 1940

One catastrophic bridge collapse every 30 years or so

See the following amazing video clip (Tacoma Narrows Bridge): http://www.enm.bris.ac.uk/research/nonlinear/tacoma/tacnarr.mpg

“... failures appear to be inevitable in the wake of prolonged success, which encourages lower margins of safety. Failures in turn lead to greater safety margins and, hence, new periods of success.”

Henry Petroski, To Engineer is Human
. . . or from “Unsinkable” Ships that Sink?

“The major difference between a thing that might go wrong and a thing that cannot possibly go wrong is that when a thing that cannot possibly go wrong goes wrong, it usually turns out to be impossible to get at or repair.”

Douglas Adams, author of *The Hitchhiker’s Guide to the Galaxy*
... or from Poorly Designed High-Tech Trains?

Train built for demonstrating magnetic levitation technology in northwest Germany rams into maintenance vehicle left on track at 200 km/h, killing 23 of 29 aboard.

Official investigation blames the accident on human error (train was allowed to depart before a clearance phone call from maintenance crew)

Not a good explanation; even low-tech trains have obstacle detection systems.

Even if manual protocol is fully adequate under normal conditions, any engineering design must take unusual circumstances into account (abuse, sabotage, terrorism).
Design Flaws in Computer Systems

Hardware example: Intel Pentium processor, 1994
For certain operands, the FDIV instruction yielded a wrong quotient
Amply documented and reasons well-known (overzealous optimization)

Software example: Patriot missile guidance, 1991
Missed intercepting a scud missile in 1st Gulf War, causing 28 deaths
Clock reading multiplied by 24-bit representation of 1/10 s (unit of time)
caused an error of about 0.0001%; normally, this would cancel out in
relative time calculations, but owing to ad hoc updates to some (not all)
calls to a routine, calculated time was off by 0.34 s (over \( \approx 100 \) hours),
during which time a scud missile travels more than 0.5 km

User interface example: Therac 25 machine, mid 1980s\(^1\)
Serious burns and some deaths due to overdose in radiation therapy
Operator entered “x” (for x-ray), realized error, corrected by entering “e”
(for low-power electron beam) before activating the machine; activation
was so quick that software had not yet processed the override

\(^1\) Accounts of the reasons vary
Causes of Human Errors in Computer Systems

1. **Personal factors (35%)**: Lack of skill, lack of interest or motivation, fatigue, poor memory, age or disability

2. **System design (20%)**: Insufficient time for reaction, tedium, lack of incentive for accuracy, inconsistent requirements or formats

3. **Written instructions (10%)**: Hard to understand, incomplete or inaccurate, not up to date, poorly organized

4. **Training (10%)**: Insufficient, not customized to needs, not up to date

5. **Human-computer interface (10%)**: Poor display quality, fonts used, need to remember long codes, ergonomic factors

6. **Accuracy requirements (10%)**: Too much expected of operator

7. **Environment (5%)**: Lighting, temperature, humidity, noise

Because “the interface is the system” (according to a popular saying), items 2, 5, and 6 (40%) could be categorized under user interface.
NO! - Bad User!!!

You've been warned 3 times that this file does not exist. Now you've made us catch this worthless exception and we're upset. Do not do this again.

OK

Big error

You really screwed up this time.

I know  What else is new  It's my first day
Properties of a Good User Interface

1. **Simplicity:** Easy to use, clean and unencumbered look

2. **Design for error:** Makes errors easy to prevent, detect, and reverse; asks for confirmation of critical actions

3. **Visibility of system state:** Lets user know what is happening inside the system from looking at the interface

4. **Use of familiar language:** Uses terms that are known to the user (there may be different classes of users, each with its own vocabulary)

5. **Minimal reliance on human memory:** Shows critical info on screen; uses selection from a set of options whenever possible

6. **Frequent feedback:** Messages indicate consequences of actions

7. **Good error messages:** Descriptive, rather than cryptic

8. **Consistency:** Similar/different actions produce similar/different results and are encoded with similar/different colors and shapes
Example from **THE RISKS DIGEST**

Forum on Risks to the Public in Computers and Related Systems
http://catless.ncl.ac.uk/Risks/
(Peter G. Neumann, moderator)

On August 17, 2006, a class-two incident occurred at the Swedish atomic reactor Forsmark. A short-circuit in the electricity network caused a problem inside the reactor and it needed to be shut down immediately, using emergency backup electricity.

However, in two of the four generators, which run on AC, the AC/DC converters died. The generators disconnected, leaving the reactor in an unsafe state and the operators unaware of the current state of the system for approximately 20 minutes.

A meltdown, such as the one in Chernobyl, could have occurred.

Coincidence of problems in multiple protection levels seems to be a recurring theme in many modern-day mishaps -- emergency systems had not been tested with the grid electricity being off
Worst Stock Market Computer Failure

April 5, 2000: Computer failure halts the trading for nearly 8 hours at the London Stock Exchange on its busiest day (end of financial year)

Firms and individual investors prevented from buying or selling stocks to minimize their capital gains taxes

Delaying end of financial year was considered, but not implemented; eventually, the system became operational at 3:45 PM and trading was allowed to continue until 8:00 PM

London Stock Exchange confirmed it had a fault in its electronic feed that sends the prices to dealers, but it gave no further explanation

A spokesman said the problems were “very technical” and involved corrupt data
Recent News Items in "THE RISKS DIGEST"

February 2012: Programming Error Doomed Russian Mars Probe
Fails to escape earth orbit due to simultaneous reboot of two subsystems

March 2012: Eighteen Companies Sued over Mobile Apps
Facebook, Apple, Twitter, and Yelp are among the companies sued over gathering data from the address books of millions of smartphone users

May 2012: Automatic Updates Considered Zombieware
Software updates take up much time/space; no one knows what’s in them

July 2012: A320 Lost 2 of 3 Hydraulic Systems on Takeoff
No loss of life; only passenger discomfort. Full account of incident not yet available, but it shows that redundancy alone is not sufficient protection

September 2013: No password Safe from New Cracking Software
A new freely available software can crack passwords of up to 55 symbols by guessing a lot of common letter combinations
How We Benefit from Failures

“When a complex system succeeds, that success masks its proximity to failure. . . . Thus, the failure of the *Titanic* contributed much more to the design of safe ocean liners than would have her success. That is the paradox of engineering and design.”

Take-Home Survey Form: Due Next Class

Personal and contact info: Name, Perm#, e-mail address, phone #(s), degrees & institutions, academic level, GPA, units completed, advisor

Main reason for taking this course

e.g.: interest, advisor’s suggestion, have to (not enough grad courses)

From the lecture topics on the course’s website, pick one topic that you believe to be most interesting

List one important fact about yourself that is not evident from your academic record or CV

e.g.: I like to solve mathematical, logical, and word puzzles

Use the space below or overleaf for any additional comments on your academic goals and/or expectations from this course
1 Background and Motivation
Droidian Slip

“You can’t keep blaming everything on human error!”

“I should get this remote control looked at.”
### STRUCTURE AT A GLANCE

| Part I — Introduction: Dependable Systems | Goals          | 1. Background and Motivation  
|                                          | Models         | 2. Dependability Attributes  
|                                          |                | 3. Combinational Modeling  
|                                          |                | 4. State-Space Modeling  
| Part II — Defects: Physical Imperfections | Methods        | 5. Defect Avoidance  
|                                          | Examples       | 6. Defect Circumvention  
|                                          |                | 7. Shielding and Hardening  
|                                          |                | 8. Yield Enhancement  
|                                          | Examples       | 10. Fault Masking  
|                                          |                | 11. Design for Testability  
|                                          |                | 12. Replication and Voting  
|                                          | Examples       | 14. Error Correction  
|                                          |                | 15. Self-Checking Modules  
|                                          |                | 16. Redundant Disk Arrays  
| Part V — Malfunctions: Architectural Anomalies | Methods | 17. Malfunction Diagnosis  
|                                          | Examples       | 18. Malfunction Tolerance  
|                                          |                | 19. Standby Redundancy  
|                                          |                | 20. Resilient Algorithms  
|                                          | Examples       | 22. Degradation Management  
|                                          |                | 23. Robust Task Scheduling  
|                                          |                | 24. Software Redundancy  
| Part VII — Failures: Computational Breaches | Methods       | 25. Failure Confinement  
|                                          | Examples       | 26. Failure Recovery  
|                                            |                | 27. Agreement and Adjudication  
|                                            |                | 28. Fail-Safe System Design  
| Appendix: Past, Present, and Future      |                |  

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Sep. 2018  
Part I – Introduction: Dependable Systems  
B Panahi  
Slide 29
1.1 The Need for Dependability

**Hardware problems**
Permanent incapacitation due to shock, overheating, voltage spike
Intermittent failure due to overload, timing irregularities, crosstalk
Transient signal deviation due to alpha particles, external interference

**Software problems**
These can also be classified as design flaws
Counter or buffer overflow
Out-of-range, unreasonable, or unanticipated input
Unsatisfied loop termination condition

Dec. 2004: “Comair runs a 15-year old scheduling software package from SBS International (www.sbsint.com). The software has a hard limit of 32,000 schedule changes per month. With all of the bad weather last week, Comair apparently hit this limit and then was unable to assign pilots to planes.”
It appears that they were using a 16-bit integer format to hold the count.

June 1996: Explosion of the Ariane 5 rocket 37 s into its maiden flight was due to a silly software error. For an excellent exposition of the cause, see:
http://www.comp.lancs.ac.uk/computing/users/dixa/teaching/CSC221/ariane.pdf
The Curse of Complexity

**Computer engineering** is the art and science of translating user requirements we do not fully understand; into hardware and software we cannot precisely analyze; to operate in environments we cannot accurately predict; all in such a way that the society at large is given no reason to suspect the extent of our ignorance.\(^1\)

**Microsoft Windows NT (1992):** \(\approx 4M\) lines of code  
**Microsoft Windows XP (2002):** \(\approx 40M\) lines of code

**Intel Pentium processor (1993):** \(\approx 4M\) transistors  
**Intel Pentium 4 processor (2001):** \(\approx 40M\) transistors  
**Intel Itanium 2 processor (2002):** \(\approx 500M\) transistors

\(^1\)Adapted from definition of structural engineering: Ralph Kaplan, *By Design: Why There Are No Locks on the Bathroom Doors in the Hotel Louis XIV and Other Object Lessons*, Fairchild Books, 2004, p. 229
Defining Failure

**Failure** is an unacceptable difference between expected and observed performance.\(^1\)

A structure (building or bridge) need not collapse catastrophically to be deemed a failure.

---

**Reasons of typical Web site failures**

- Hardware problems: 15%
- Software problems: 34%
- Operator error: 51%

---

\(^1\) Definition used by the Tech. Council on Forensic Engineering of the Amer. Society of Civil Engineers
How the customer explained it

How the Project Leader understood it

How the Analyst designed it

How the Programmer wrote it

How the Business Consultant described it

How the project was documented

What operations were installed

How the customer was billed

How it was supported

What the customer really needed
Design Flaws: “To Engineer is Human”\textsuperscript{1}

Complex systems almost certainly contain multiple design flaws

Redundancy in the form of safety factor is routinely used in buildings and bridges

Example of a more subtle flaw:
Disney Concert Hall in Los Angeles reflected light into nearby building, causing discomfort for tenants due to blinding light and high temperature

\textsuperscript{1} Title of book by Henry Petroski
Why Dependability Is a Concern

Reliability of $n$-transistor system, each having failure rate $\lambda$

$$R(t) = e^{-n\lambda t}$$

There are only 3 ways of making systems more reliable

**Reduce $\lambda$**

**Reduce $n$**

**Reduce $t$**

**Alternative:**
Change the reliability formula by introducing redundancy in system
The Three Principal Arguments

The reliability argument

\[ \lambda = 10^{-9} \text{ per transistor per hour} \]

Reliability formula \[ R(t) = e^{-n \lambda t} \]

The on-board computer of a 10-year unmanned space mission can contain only \( O(10^3) \) transistors if the mission is to have a 90% success probability.

The safety argument

Airline’s risk: \( O(10^3) \) planes \( \times O(10^2) \) flights \( \times 10^{-2} \) computer failures / 10 hr \( \times 0.001 \) crash/failure \( \times O(10^2) \) deaths \( \times O($10^7)/death = $ \) billions / yr.

The availability argument

A central phone facility’s down time should not exceed a few minutes / yr.

Mean time to failure: \( \text{MTTF} = 1/(n \lambda) \rightarrow \)

Components \( n = O(10^4) \), if we need 20-30 min for diagnosis and repair.

Fig. 1.1
Learning Curve: “Normal Accidents”

Example: Risk of piloting a plane

1903  First powered flight
1908  First fatal accident
1910  Fatalities = 32 (≈2000 pilots worldwide)
1918  US Air Mail Service founded
      Pilot life expectancy = 4 years
      31 of the first 40 pilots died in service
1922  One forced landing for every 20 hours of flight

Today  Commercial airline pilots pay normal life insurance rates

Unfortunately, the learning curve for computers and computer-based systems is not as impressive

1 Title of book by Charles Perrow (Ex. p. 125)
Mishaps, Accidents, and Catastrophes

**Mishap:** misfortune; unfortunate accident

**Accident:** unexpected (no-fault) happening causing loss or injury

**Catastrophe:** final, momentous event of drastic action; utter failure

At one time (following the initial years of highly unreliable hardware), computer mishaps were predominantly the results of human error

Now, most mishaps are due to complexity (unanticipated interactions)

---

Rube Goldberg contraptions

The butterfly effect
1.2 A Motivating Case Study

Data availability and integrity concerns
Distributed DB system with 5 sites
Full connectivity, dedicated links
Only direct communication allowed
Sites and links may malfunction
Redundancy improves availability

\[ S: \text{Probability of a site being available} \]
\[ L: \text{Probability of a link being available} \]

Single-copy availability \( = SL \)
Unavailability \( = 1 – SL \)
\[ = 1 – 0.99 \times 0.95 = 5.95\% \]

Data replication methods, and a challenge
File duplication: home / mirror sites
File triplication: home / backup 1 / backup 2
Are there availability improvement methods with less redundancy?

Fig. 1.2
Data Duplication: Home and Mirror Sites

\[ A = SL + (1 - SL)SL \]

Primary site can be reached
Primary site inaccessible

Duplicated availability = \( 2SL - (SL)^2 \)
Unavailability = \( 1 - 2SL + (SL)^2 \)
\[ = (1 - SL)^2 = 0.35\% \]

Data unavailability reduced from 5.95% to 0.35%

Availability improved from \( \approx 94\% \) to 99.65%

Fig. 1.2
Data Triplication: Home and Two Backups

S: Site availability  e.g., 99%
L: Link availability  e.g., 95%

\[ A = SL + (1 - SL)SL + (1 - SL)^2SL \]

Primary site can be reached  Backup 1 can be reached  Backup 2 can be reached
Primary site inaccessible  Primary and backup 1 inaccessible

Triplicated avail. = 3SL – 3(SL)^2 – (SL)^3
Unavailability = 1 – 3SL – 3(SL)^2 + (SL)^3
= (1 – SL)^3 = 0.02%

Data unavailability reduced from 5.95% to 0.02%

Availability improved from \( \approx 94\% \) to 99.98%

Fig. 1.2
Data Dispersion: Three of Five Pieces

\[ A = (SL)^4 + 4(1 - SL)(SL)^3 + 6(1 - SL)^2(SL)^2 \]

- All 4 pieces can be reached
- Exactly 3 pieces can be reached
- Only 2 pieces can be reached

\( S \): Site availability  
\( L \): Link availability

**Dispersed avail.**  
\[ 6(SL)^2 - 8(SL)^3 + 3(SL)^4 \]

**Availability** = 99.92%  
**Unavailability** = 1 – Availability = 0.08%

### Scheme

<table>
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<th>Nonredund.</th>
<th>Duplication</th>
<th>Triplication</th>
<th>Dispersion</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unavailability</td>
<td>5.95%</td>
<td>0.35%</td>
<td>0.02%</td>
<td>0.08%</td>
</tr>
<tr>
<td>Redundancy</td>
<td>0%</td>
<td>100%</td>
<td>200%</td>
<td>67%</td>
</tr>
</tbody>
</table>
Dispersion for Data Security and Integrity

$f(x) = ax^2 + bx + c$

Encoding with 67% redundancy

Note that two pieces would be inadequate for reconstruction

Fig. 1.2
Questions Ignored in Our Simple Example

1. How redundant copies of data are kept consistent
When a user modifies the data, how to update the redundant copies (pieces) quickly and prevent the use of stale data in the meantime?

2. How malfunctioning sites and links are identified
Malfunction diagnosis must be quick to avoid data contamination

3. How recovery is accomplished when a malfunctioning site/link returns to service after repair
The returning site must be brought up to date with regard to changes

4. How data corrupted by the actions of an adversary is detected
This is more difficult than detecting random malfunctions

The example does demonstrate, however, that:
- Many alternatives are available for improving dependability
- Proposed methods must be assessed through modeling
- The most cost-effective solution may be far from obvious
1.3 Impairments to Dependability

- Flaw
- Fault
- Error
- Hazard
- Bug
- Degradation
- Defect
- Failure
- Intrusion
- Malfunction
- Crash
The Fault-Error-Failure Cycle

Fig. 1.3

Schematic diagram of the Newcastle hierarchical model and the impairments within one level.
The Four-Universe Model

<table>
<thead>
<tr>
<th>Universe</th>
<th>Impairment</th>
</tr>
</thead>
<tbody>
<tr>
<td>Physical</td>
<td>Failure</td>
</tr>
<tr>
<td>↓</td>
<td>↓</td>
</tr>
<tr>
<td>Logical</td>
<td>Fault</td>
</tr>
<tr>
<td>↓</td>
<td>↓</td>
</tr>
<tr>
<td>Informational</td>
<td>Error</td>
</tr>
<tr>
<td>↓</td>
<td>↓</td>
</tr>
<tr>
<td>External</td>
<td>Crash</td>
</tr>
</tbody>
</table>

Cause-effect diagram for Avižienis’ four-universe model of impairments to dependability.
Unrolling the Fault-Error-Failure Cycle

Cause-effect diagram for an extended six-level view of impairments to dependability.
1.4 A Multilevel Model

Legend:
- Ideal
- Defective
- Faulty
- Erroneous
- Malfunctioning
- Degraded
- Failed

Device
Logic
State
System
Service
Result

Fig. 1.6
1.5 Examples and Analogies

Example 1.4: Automobile brake system

<table>
<thead>
<tr>
<th>Type</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Defect</td>
<td>Brake fluid piping has a weak spot or joint</td>
</tr>
<tr>
<td>Fault</td>
<td>Brake fluid starts to leak out</td>
</tr>
<tr>
<td>Error</td>
<td>Brake fluid pressure drops too low</td>
</tr>
<tr>
<td>Malfunction</td>
<td>Braking force is below expectation</td>
</tr>
<tr>
<td>Degradation</td>
<td>Braking requires higher force or takes longer</td>
</tr>
<tr>
<td>Failure</td>
<td>Vehicle does not slow down or stop in time</td>
</tr>
</tbody>
</table>

Note in particular that not every defect, fault, error, malfunction, or degradation leads to failure.
An analogy for our multi-level model of dependable computing. Defects, faults, errors, malfunctions, degradations, and failures are represented by pouring water from above. Valves represent avoidance and tolerance techniques. The goal is to avoid overflow.

Fig. 1.7

Opening drain valves represents tolerance techniques
1.6 Dependable Computer Systems

**Long-life systems:** Fail-slow, Rugged, High-reliability
Spacecraft with multiyear missions, systems in inaccessible locations
Methods: Replication (spares), error coding, monitoring, shielding

**Safety-critical systems:** Fail-safe, Sound, High-integrity
Flight control computers, nuclear-plant shutdown, medical monitoring
Methods: Replication with voting, time redundancy, design diversity

**Non-stop systems:** Fail-soft, Robust, High-availability
Telephone switching centers, transaction processing, e-commerce
Methods: HW/info redundancy, backup schemes, hot-swap, recovery

Just as performance enhancement techniques gradually migrate from supercomputers to desktops, so too dependability enhancement methods find their way from exotic systems into personal computers.
2 Dependability Attributes
THE PROBLEM IS YOUR MODEM CAN'T INTERFACE WITH YOUR ISP BECAUSE YOUR RJ 11 CABLE NEEDS UPGRADING

WILL IT COST MUCH?

THAT DEPENDS ON WHETHER YOU KNOW I JUST SAID "YOU NEED A LONGER PHONE CORD"

"Every time we successfully recover from a technical problem, the computer likes a high five."

"That foul smell is coming from your computer. You've got some old data in there that's gone bad."

Sep. 2018 Part I – Introduction: Dependable Systems
### STRUCTURE AT A GLANCE

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Appendix: Past, Present, and Future

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Part I – Introduction: Dependable Systems  
Slide 55
2.1 Aspects of Dependability

The -ilities

Serviceability

Reliability
Reliability, MTTF = MTFF

Security

Safety
Risk, consequence

Resilience

Testability
Controllability, observability

Maintainability

Robustness

Integrity

Availability
Pointwise av., Interval av., MTBF, MTTR

Performability
Performability, MCBF
Concepts from Probability Theory

**Probability density function:** $pdf$

$$f(t) = \text{prob}[t \leq x \leq t + dt] / dt = dF(t) / dt$$

**Cumulative distribution function:** $CDF$

$$F(t) = \text{prob}[x \leq t] = \int_0^t f(x) \, dx$$

**Expected value of $x$**

$$E_x = \int_{-\infty}^{+\infty} x f(x) \, dx = \sum_k x_k f(x_k)$$

**Variance of $x$**

$$\sigma_x^2 = \int_{-\infty}^{+\infty} (x - E_x)^2 f(x) \, dx$$

$$= \sum_k (x_k - E_x)^2 f(x_k)$$

**Covariance of $x$ and $y$**

$$\psi_{x,y} = E[(x - E_x)(y - E_y)]$$

$$= E[xy] - E_x E_y$$
Some Simple Probability Distributions

- Uniform
- Exponential
- Normal
- Binomial
Layers of Safeguards

With multiple layers of safeguards, a system failure occurs only if warning symptoms and compensating actions are missed at every layer, which is quite unlikely

Is it really?

The computer engineering literature is full of examples of mishaps when two or more layers of protection failed at the same time

Multiple layers increase the reliability significantly only if the “holes” in the representation above are fairly randomly and independently distributed, so that the probability of their being aligned is negligible

Dec. 1986: ARPANET had 7 dedicated lines between NY and Boston; A backhoe accidentally cut all 7 (they went through the same conduit)
2.2 Reliability and MTTF

**Reliability:** \( R(t) \)

Probability that system remains in the “Good” state through the interval \([0, t]\)

\[
R(t + dt) = R(t) [1 - z(t) dt]
\]

Hazard function

\[
R(t) = 1 - F(t) \quad \text{CDF of the system lifetime, or its unreliability}
\]

Constant hazard function \( z(t) = \lambda \Rightarrow R(t) = e^{-\lambda t} \)

(system failure rate is independent of its age)

**Mean time to failure:** MTTF

\[
\text{MTTF} = \int_0^{+\infty} t f(t) dt = \int_0^{+\infty} R(t) dt
\]

Expected value of lifetime

Area under the reliability curve (easily provable)
Failure Distributions of Interest

**Exponential:** $z(t) = \lambda$

\[
R(t) = e^{-\lambda t} \quad \text{MTTF} = 1/\lambda
\]

**Rayleigh:** $z(t) = 2\lambda(\lambda t)$

\[
R(t) = e^{(-\lambda t)^2} \quad \text{MTTF} = \left(1/\lambda \right) \sqrt{\pi} / 2
\]

**Weibull:** $z(t) = \alpha \lambda (\lambda t)^{\alpha-1}$

\[
R(t) = e^{(-\lambda t)^\alpha} \quad \text{MTTF} = \left(1/\lambda \right) \Gamma(1 + 1/\alpha)
\]

**Erlang:**

Gen. exponential \hspace{1cm} \text{MTTF} = k/\lambda

**Gamma:**

Gen. Erlang

(becomes Erlang for $b$ an integer)

**Normal:**

Reliability and MTTF formulas are complicated

**Discrete versions**

**Geometric**

\[
R(k) = q^k
\]

**Discrete Weibull**

**Binomial**

**Discrete versions**

**Binomial**
Elaboration on Weibull Distribution

**Weibull:** 
\[ z(t) = \alpha \lambda (\lambda t)^{\alpha-1} \]
\[ R(t) = e^{(-\lambda t)^\alpha} \]

\[ \ln \ln[1/R(t)] = \alpha (\ln t + \ln \lambda) \]

- \( \alpha < 1 \), Infant mortality
- \( \alpha = 1 \), Constant hazard rate (exponential)
- \( 1 < \alpha < 4 \), Rising hazard (fatigue, corrosion)
- \( \alpha > 4 \), Rising hazard (rapid wearout)

The following diagrams were taken from http://www.rpi.edu/~albenr/presentations/Reliability.ppt which is no longer available.
Comparing Reliabilities

**Reliability difference:** \( R_2 - R_1 \)

**Reliability gain:** \( R_2 / R_1 \)

**Reliability improvement factor**
\[ \text{RIF}_{2/1} = \frac{[1-R_1(t_M)]}{[1-R_2(t_M)]} \]

Example:
\[ \frac{[1 - 0.9]}{[1 - 0.99]} = 10 \]

**Reliability improv. index**
\[ \text{RII} = \log R_1(t_M) / \log R_2(t_M) \]

**Mission time extension**
\[ \text{MTE}_{2/1}(r_G) = T_2(r_G) - T_1(r_G) \]

**Mission time improv. factor:**
\[ \text{MTIF}_{2/1}(r_G) = T_2(r_G) / T_1(r_G) \]
Analog of Amdahl’s Law for Reliability

Amdahl’s law: If in a unit-time computation, a fraction $f$ doesn’t change and the remaining fraction $1 - f$ is speeded up to run $p$ times as fast, the overall speedup will be $s = 1 / (f + (1 - f)/p)$

Consider a system with two parts, having failure rates $\phi$ and $\lambda - \phi$

Improve the failure rate of the second part by a factor $p$, to $(\lambda - \phi)/p$

$$R_{\text{original}} = \exp(-\lambda t) \quad R_{\text{improved}} = \exp[-(\phi + (\lambda - \phi)/p)t]$$

**Reliability improv. index**

$$RII = \log R_{\text{original}} / \log R_{\text{improved}}$$

$$RII = \lambda / (\phi + (\lambda - \phi)/p)$$

Letting $\phi / \lambda = f$, we have: $RII = 1 / (f + (1 - f)/p)$

See B. Parhami’s paper in July 2015

*IEEE Computer*
2.3 Availability, MTTR, and MTBF

(Interval) Availability: \( A(t) \)
Fraction of time that system is in the “Up” state during the interval \([0, t]\)

Steady-state availability: \( A = \lim_{t \to \infty} A(t) \)

Pointwise availability: \( a(t) \)
Probability that system available at time \( t \)
\[ A(t) = \frac{1}{t} \int_0^t a(x) \, dx \]

Availability = Reliability, when there is no repair

Availability is a function not only of how rarely a system fails (reliability) but also of how quickly it can be repaired (time to repair)

\[ A = \frac{MTTF}{MTTF + MTTR} = \frac{MTTF}{MTBF} = \frac{\mu}{\lambda + \mu} \]

Repair rate \( 1/\mu = MTTR \)
(Will justify this equation later)

In general, \( \mu \gg \lambda \), leading to \( A \approx 1 \)
System Up and Down Times

Short repair time implies good **maintainability (serviceability)**

![Diagram showing system up and down times with time to first failure, time between failures, and repair time.]

**Fig. 2.6**
2.4 Performability and MCBF

**Performability: \( P \)**  
Composite measure, incorporating both performance and reliability

**Simple example**  
Worth of “Up2” twice that of “Up1”  
\( p_{\text{Up}_i} = \) probability system is in state \( \text{Up}_i \)

\[
P = 2p_{\text{Up}_2} + p_{\text{Up}_1}
\]

\( p_{\text{Up}_2} = 0.92, \quad p_{\text{Up}_1} = 0.06, \quad p_{\text{Down}} = 0.02, \quad P = 1.90 \)  
(system performance equiv. To that of 1.9 processors on average)

Performability improvement factor of this system (akin to RIF) relative to a fail-hard system that goes down when either processor fails:  
\[
\text{PIF} = \frac{2 - 2 \times 0.92}{2 - 1.90} = 1.6
\]

**Question:** What is system availability here?
System Up, Partially Up, and Down Times

Important to prevent direct transitions to the “Down” state (coverage)

Start state

Up 2
Partial failure

Up 1
Failure

Partial repair

Down

Fig. 2.8

Time

0  t_1  t_2  t'_2  t'_1  t_3  t'_3  t

Up

Partially Up

Down

Partial Failure

Total Failure

Partial Repair

MCBF

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2.5 Integrity and Safety

**Integrity and safety are similar**
Integrity is inward-looking: capacity to protect system resources (e.g., data)
Safety is outward-looking: consequences of incorrect actions to users

**A high-integrity system is robust**
Data is not corrupted by low-severity causes

Safety is distinct from reliability: a fail-safe system may not be very reliable in the traditional sense
Basic Safety Assessment

**Risk:** Prob. of being in “Unsafe Down” state
There may be multiple unsafe states, each with a different consequence (cost)

**Simple analysis**
Lump “Safe Down” state with “Up” state; proceed as in reliability analysis

**More detailed analysis**
Even though “Safe Down” state is more desirable than “Unsafe Down”, it is still not as desirable as the “Up” state; so keeping it separate makes sense

We may have multiple unsafe states
Quantifying Safety

Risk = Frequency × Magnitude

Consequence / Unit time × Events / Unit time × Consequence / Event

Risk = Probability × Severity

Magnitude or severity is measured in some suitable unit (say, dollars)

When there are multiple unsafe outcomes, the probability of each is multiplied by its severity (cost) and the results added up
Safety Assessment with More Transitions

If a repair transition is introduced between “Safe Down” and “Up” states, we can tackle questions such as the expected outage of the system in safe mode, and thus its availability.

**Modeling safety procedures**
A safe failure can become unsafe or an unsafe failure can turn into a more severe safety problem due to mishandling or human error.

This can be easily modeled by adding appropriate transitions.

---

*Fig. 2.10*

Three-state fail-safe system

Start state → Safe Down

- Repair → Up
- Failure → Unsafe Down

Mishandling
Fallacies of Risk*

1. *Sheer size*: X is accepted. Y is a smaller risk than X. ∴ Y should be accepted.
2. *Converse sheer size*: X is not accepted. Y is a larger risk than X. ∴ Y should not be accepted.
3. *Naturalness*: X is natural. ∴ X should be accepted.
4. *Ostrich’s*: X has no detectable risk. ∴ X has no unacceptable risks.
5. *Proof-seeking*: There is no scientific proof that X is dangerous. ∴ No action should be taken against X.
6. *Delay*: If we wait, we will know more about X. ∴ No decision about X should be made now.
7. *Technocratic*: It is a scientific issue how dangerous X is. ∴ Scientists should decide whether or not X is acceptable.
8. *Consensus*: We must ask the experts about X. ∴ We must ask the experts about a consensus opinion on X
9. *Pricing*: We have to weigh the risk of X against its benefits. ∴ We must put a price on the risk of X
10. *Infallibility*: Experts and the public do not have the same attitude about X. ∴ The public is wrong about X

---

2.6 Privacy and Security

Privacy and security impairments are human-related
Accidental: operator carelessness, improper reaction to safety warnings
Malicious attacks: Hackers, viruses, and the like

Privacy is compromised when
confidential or personal data are disclosed to unauthorized parties

Security is breached when
account information in a bank is improperly modified, say

Security is distinct from both reliability and safety: a system that automatically locks up when a security breach is suspected may not be very reliable or safe in the traditional sense
Quantifying Security

In theory, security can be quantified in the same way as safety:

\[
\text{Risk} = \text{Frequency} \times \text{Magnitude}
\]

\[
\text{Risk} = \text{Probability} \times \text{Severity}
\]

But because security breaches are often not accidental, they are ill-suited to probabilistic treatment.
3 Combinational Modeling
“We installed little monitors because they make all of our problems look smaller.”

When model does not match reality.
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Appendix: Past, Present, and Future
3.1 Modeling by Case Analysis

Revisiting the motivating example: Data files to be stored on five sites so that they remain available despite site and link malfunctions.

\( S = \) Site availability \((a_S \text{ in textbook})\)
\( L = \) Link availability \((a_L \text{ in textbook})\)

Some possible strategies:
- Duplication on home site and mirror site
- Triplication on home site and 2 backups
- Data dispersion through coding

Here, we ignore the important problem of keeping the replicas consistent and do not worry about malfunction detection and attendant recovery actions.

Five-site distributed computer system
Data Availability with Home and Mirror Sites

Assume data file must be obtained directly from a site that holds it

\[ A = SL + (1 - SL)SL = 2SL - (SL)^2 \]

For example, \( S = 0.99, \ L = 0.95, \ A = 0.9965 \)
With no redundancy, \( A = 0.99 \times 0.95 = 0.9405 \)

**Combinational modeling:**
Consider all combinations of circumstances that lead to availability/success (unavailability/failure)

Analysis by considering mutually exclusive subcases
Data Availability with Triplication

\[ A = SL + (1 - SL)SL + (1 - SL)^2SL \]
\[ = 3SL - 3(SL)^2 + (SL)^3 \]

For example, \( S = 0.99, L = 0.95, A = 0.9998 \)
With duplication, \( A = 0.9965 \)
With no redundancy, \( A = 0.9405 \)

Can merge these two cases

\[ A = SL + (1 - SL)[SL + (1 - SL)SL] \]
Data Availability with File Dispersion

Encode an $l$-bit file into $\approx 5l/3$ bits (67% redund.)
Break encoded file into 5 pieces of length $l/3$
Store each piece on one of the 5 sites
Any 3 of the 5 pieces can be used to reconstruct the original file

File accessible if 2 out of 4 sites accessible

$$A = (SL)^4 + 4(1 - SL)(SL)^3 + 6(1 - SL)^2(SL)^2$$
$$= 6(SL)^2 - 8(SL)^3 + 3(SL)^4$$

For example, $S = 0.99, L = 0.95, A = 0.9992$, Redundancy = 67%
With duplication, $A = 0.9965$, Redundancy = 100%
With triplication, $A = 0.9998$, Redundancy = 200%
With no redundancy, $A = 0.9405$
3.2 Series and Parallel Systems

A series system is composed of $n$ units all of which must be healthy for the system to function properly:

$$R = \prod R_i$$

**Example:** Redundant system of valves in series with regard to stuck-on-shut malfunctions (tolerates stuck-on-open valves)

**Example:** Redundant system of valves in parallel with regard to stuck-on-open malfunctions (tolerates stuck-on-shut valves)
Series System: Implications to Design

Assume exponential reliability law

\[ R_i = \exp[-\lambda_i t] \]

\[ R = \prod R_i = \exp[-(\sum \lambda_i) t] \]

Given the reliability goal \( r \), find the required value of \( \sum \lambda_i \)

Assign a failure rate “budget” to each unit and proceed with its design

May have to reallocate budgets if design proves impossible or costly
Parallel System

A parallel system is composed of $n$ units, the health of one of which is enough for proper system operation.

$$1 - R = \prod (1 - R_i) \rightarrow R = 1 - \prod (1 - R_i)$$

That is, the system fails only if all units malfunction.

**Example:** Redundant system of valves in parallel with regard to stuck-on-shut malfunctions (tolerates stuck-on-shut valves).

**Example:** Redundant system of valves in series with regard to stuck-on-open malfunctions (tolerates stuck-on-open valves).
Parallel System: Implications to Design

Assume exponential reliability law

\[ R_i = \exp[-\lambda_i t] \]

\[ 1 - R = \prod (1 - R_i) \]

Given the reliability goal \( r \), find the required value of \( 1 - r = \prod (1 - R_i) \)

Assign a failure probability “budget” to each unit

For example, with identical units, \( 1 - R_m = \sqrt[n]{1 - r} \)

Assume \( r = 0.9999 \), \( n = 4 \) \( \Rightarrow 1 - R_m = 0.1 \) (module reliability must be 0.9)

Conversely, for \( r = 0.9999 \) and \( R_m = 0.9 \), \( n = 4 \) is needed
The Perils of Modeling

An example two-way parallel system:
In a passenger plane, the failure rate of the cabin pressurizing system is $10^{-5}$/hr (loss of cabin pressure occurs once per $10^5$ hours of flight)

Failure rate of the oxygen-mask deployment system is also $10^{-5}$/hr

Assuming failure independence, both systems fail at a rate of $10^{-10}$/hr

Fatality probability for a 10-hour flight is about $10^{-10} \times 10 = 10^{-9}$
(10$^{-9}$ or less is generally deemed acceptable)

Probability of death in a car accident is $\approx 1/6000$ per year ($>10^{-7}$/hr)

Alternate reasoning
Probability of cabin pressure system failure in 10-hour flight is $10^{-4}$
Probability of oxygen masks failing to deploy in 10-hour flight is $10^{-4}$
Probability of both systems failing in 10-hour flight is $10^{-8}$
Why is this result different from that of our earlier analysis ($10^{-9}$)?
Which one is correct?
When we multiply the two per-hour failure rates and then take the flight duration into account, we are assuming that only the failure of the two systems within the same hour is catastrophic.

This produces an optimistic reliability estimate \(1 - 10^{-9}\).

When we multiply the two flight-long failure rates, we are assuming that the failure of these systems would be catastrophic at any time.

This produces a pessimistic reliability estimate \(1 - 10^{-8}\).
The Concept of Coverage

For $r = 0.9999$ and $R_i = 0.9$, $n = 4$ is needed

Standby sparing: One unit works; others are also active concurrently or they may be inactive (spares)

When a malfunction of the main unit is detected, it is removed from service and an alternate unit is brought on-line; our analysis thus far assumes perfect malfunction detection and reconfiguration

$$R = 1 - (1 - R_m)^n = R_m \frac{1 - (1 - R_m)^n}{1 - (1 - R_m)}$$

Let the probability of correct malfunction detection and successful reconfiguration be $c$ (coverage factor, $c < 1$)

$$R = R_m \frac{1 - c^n(1 - R_m)^n}{1 - c(1 - R_m)}$$

See [Siew92], p. 288
Impact of Coverage on System Reliability

$c$: prob. of correct malfunction detection and successful reconfiguration

\[ R = R_m \frac{1 - c^n(1 - R_m)^n}{1 - c(1 - R_m)} \]

Assume $R_m = 0.95$
Plot $R$ as a function of $n$ for $c = 0.9, 0.95, 0.99, 0.999, 0.9999, 1$

Unless $c$ is near-perfect, adding more spares has no significant effect on reliability

In practice $c$ is not a constant and may deteriorate with more spares; so too many spares may be detrimental to reliability
3.3 Classes of $k$-out-of-$n$ Systems

There are $n$ modules, any $k$ of which are adequate for proper system functioning.

**Example:** System with 2-out-of-3 voting

Assume perfect voter

$$R = R_1 R_2 R_3 + R_1 R_2 (1 - R_3) + R_2 R_3 (1 - R_1) + R_3 R_1 (1 - R_2)$$

With all units having the same reliability $R_m$ and imperfect voter:

$$R = (3R_m^2 - 2R_m^3) R_v$$

Triple-modular redundancy (TMR)

$$R = \sum_{j=k}^{n} \binom{n}{j} R_m^j (1 - R_m)^{n-j}$$

$k$-out-of-$n$ system in general

Assuming that any 2 malfunctions in TMR lead to failure is pessimistic.

With binary outputs, we can model compensating errors (when two malfunctioning modules produce 0 and 1 outputs).
**n-Modular Redundancy with Replicated Voters**

Voters (all but the final one in a chain) no longer critical components

Can model as a series system of 2-out-of-3 subsystems
Consecutive $k$-out-of-$n$:G ($k$-out-of-$n$:F) System

Units are ordered and the functioning (failure) of $k$ consecutive units leads to proper system function (system failure)

Ordering may be linear (usual case) or circular

Example: System of street lights may be considered a consecutive 2-out-of-$n$:F system

Example: The following redundant bus reconfiguration scheme is a consecutive 2-out-of-4:G system

- Common control for shift-switch settings: up, straight, or down
- From module
- Redundant bus lines
3.4 Reliability Block Diagrams

The system functions properly if a string of healthy units connect one side of the diagram to the other

\[ 1 - R = (1 - R_1 R_2) (1 - R_3 R_4) \]

**Example:** Parallel connection of series pairs of valves (tolerates one stuck-on-shut and one stuck-on-open valve)

**Example:** Series connection of parallel pairs of valves (tolerates one stuck-on-shut and one stuck-on-open valve)

\[ R = [1 - (1 - R_1)(1 - R_3)] \times [1 - (1 - R_2)(1 - R_4)] \]
Non-Series/Parallel Systems

The system functions properly if a string of healthy units connect one side of the diagram to the other.

We can think of Unit 5 as being able to replace Units 2 and 3.

\[ R = R_3 \times \text{prob(system OK | Unit 3 OK)} \]
\[ + (1 - R_3) \times \text{prob(system OK | Unit 3 not OK)} \]

\[ R_{3\text{OK}} = [1 - [R_1(1 - (1 - R_2)(1 - R_5))])(1 - R_6)] R_4 \]

\[ R_{3\neg\text{OK}} = R_1 R_5 R_4 \]
Analysis Using Success Paths

\[ R \leq 1 - \prod_i (1 - R_{i{th} \text{ success path}}) \]

This yields an upper bound on reliability because it considers the paths to be independent.

\[ R \leq 1 - (1 - R_1 R_5 R_4) (1 - R_1 R_2 R_3 R_4)(1 - R_6 R_3 R_4) \]

[*]

With equal module reliabilities:

\[ R \leq 1 - (1 - R_m)^2 (1 - R_m^4) \]

If we expand [*] by multiplying out, removing any power for the various reliabilities, we get an exact reliability expression:

\[ R = 1 - (1 - R_1 R_4 R_5)(1 - R_3 R_4 R_6 - R_1 R_2 R_3 R_4 - R_1 R_2 R_3 R_4 R_5 - R_1 R_2 R_3 R_4 R_6 - R_1 R_2 R_3 R_4 R_5 R_6) \]

\[ = R_3 R_4 R_6 + R_1 R_2 R_3 R_4 + R_1 R_2 R_3 R_4 R_6 + R_1 R_4 R_5 - R_1 R_3 R_4 R_5 R_6 \]

\[ - R_1 R_2 R_3 R_4 R_5 - R_1 R_2 R_3 R_4 R_5 R_6 \]  

(Verify for the case of equal \( R_j \))
3.5 Reliability Graphs

A reliability graph is a schematic representation of system components, their interactions, and their roles in proper system operation.

Use generalized series-parallel connections to visualize success paths, which are directed paths from a source node to a sink node (both unique).

Each module name labels one edge: module failure = edge disconnect.

An edge labeled \( \infty \) is never disconnected.
3.6 The Fault-Tree Method

Top-down approach to failure analysis:

Start at the top (tree root) with an undesirable event called a “top event” and then determine all the possible ways that the top event can occur.

Analysis proceeds by determining how the top event can be caused by individual or combined lower-level undesirable events.

**Example:**

Top event is “being late for work”
Clock radio not turning on, family emergency, bus not running on time
Clock radio won’t turn on if there is a power failure and battery is dead.


Fault tree handbook:
Fault Tree Analysis: The Process

1. Identify “top event”

2. Identify -level contributors to top event

3. Use logic gate to connect 1\textsuperscript{st} level to top

4. Identify 2\textsuperscript{nd}-level contributors

5. Link 2\textsuperscript{nd} level to 1\textsuperscript{st} level

6. Repeat until done

Other symbols:
- XOR (not used in reliability analysis)
- \(k/n\) gate
- Enabling condition
- Inhibit gate
- External event

Basic events (leaf, atomic)
Composite events
A cut set is any set of initiators so that the failure of all of them induces the top event.

Minimal cut set: A cut set for which no subset is also a cut set.

Minimal cut sets for this example: \{a, b\}, \{a, d\}, \{b, c\}

Just as logic circuits can be transformed to different (simpler) ones, fault trees can be manipulated to obtain equivalent forms.

Path set: Any set of initiators so that if all are failure-free, the top event is inhibited. (to derive path sets, exchange AND gates and OR gates and then find cut sets)

What are the path sets for this example?
Converting Fault Trees to Reliability Block Diagrams

Minimal cut sets for this example:
\{a, b\}, \{a, d\}, \{b, c\}

Another example:
Minimal cut set \{a, b\}, \{a, c\}, \{a, d\}, \{c, d, e, f\}

Construct a fault tree for the above
Derive a reliability block diagram
What are the path sets for this example?

Applications of cut sets:
1. Evaluation of reliability
2. Common-cause failure assessment
3. Small cut set \(\rightarrow\) high vulnerability
Hierarchy of Combinational Models

Fault trees with repeated elements

Reliability graphs

Fault trees with no repetition

Reliability block diagrams
4 State-Space Modeling
"It's the latest innovation in office safety. When your computer crashes, an air bag is activated so you won't bang your head in frustration."

"An amazing thing happened at work today. For 8 minutes, my computer and I were both functional at the same time!"

"Can't you do anything right?"

"Accidentally listened to my motivation tapes backwards and became a failure. Please help."

### STRUCTURE AT A GLANCE

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Appendix: Past, Present, and Future

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What Is State-Space Modeling?

With respect to availability of resources and computational capabilities, a system can be viewed as being in one of several possible states.

The number of states can be large, if we want to make fine distinctions, or it can be relatively small if we lump similar states together.

**State transitions:**
System moves from one state to another as resource availability and computational power change due to various events.

State-space modeling entails quantifying transition probabilities so as to determine the probability of the system being in each state; from this, we derive reliability, availability, safety, and other desired parameters.
4.1 Markov Chains and Models

Represented by a state diagram with transition probabilities
Sum of all transition probabilities out of each state is 1

The state of the system is characterized by the vector \( (s_0, s_1, s_2, s_3) \)
(1, 0, 0, 0) means that the system is in state 0 
(0.5, 0.5, 0, 0) means that the system is in state 0 or 1 with equal prob’s
(0.25, 0.25, 0.25, 0.25) represents complete uncertainty

Transition matrix: \( M = \begin{pmatrix} 0.3 & 0.4 & 0.3 & 0 \\ 0.5 & 0.4 & 0 & 0.1 \\ 0 & 0.2 & 0.7 & 0.1 \\ 0.4 & 0 & 0.3 & 0.3 \end{pmatrix} \)
Markov matrix (rows sum to 1)

Example:
\( (s_0, s_1, s_2, s_3) = (0.5, 0.5, 0, 0) M = (0.4, 0.4, 0.15, 0.05) \)
\( (s_0, s_1, s_2, s_3) = (0.4, 0.4, 0.15, 0.05) M = (0.34, 0.365, 0.225, 0.07) \)
Stochastic Sequential Machines

Transition taken from state $s$ under input $j$ is not uniquely determined. Rather, a number of states may be entered with different probabilities.

There will be a separate transition (Markov) matrix for each input value.

Transitions, $j = 0$: $M = \begin{pmatrix} 0.3 & 0.4 & 0.3 & 0 \\ 0.5 & 0.4 & 0 & 0.1 \\ 0 & 0.2 & 0.7 & 0.1 \\ 0.4 & 0 & 0.3 & 0.3 \end{pmatrix}$

Transitions, $j = 1$: $M = \begin{pmatrix} 0.5 & 0.2 & 0.1 & 0.2 \\ 0.1 & 0.4 & 0.4 & 0.1 \\ 0.3 & 0 & 0.2 & 0.5 \\ 0.2 & 0.6 & 0 & 0.2 \end{pmatrix}$

A Markov chain can be viewed as a stochastic sequential machine with no input.
Sample Applications of Markov Modeling

“Hidden Markov Model” for recognition problems

Figure 6: Partial Markov model of trips made between home, Centennial Research Building (CRB), and Dept. of Veterans Affairs (VA). Because some paths are not shown, the ratios do not sum to 1.
Merging States in a Markov Model

There are three identical units
1 = Unit is up
0 = Unit is down

All solid lines λ
Dashed lines μ

Simpler equivalent model for 3-unit fail-soft system

Whether or not states are merged depends on the model’s semantics

Failed state if TMR
4.2 Modeling Nonrepairable Systems

Rate of change for the probability of being in state 1 is \(-\lambda\)

\[
p'_1 = -\lambda p_1 \\
p_1 + p_0 = 1
\]

Reliability as a function of time:

\[
R(t) = p_1(t) = e^{-\lambda t}
\]

Two-state system:

- The label \(\lambda\) on this transition means that over time \(dt\), the transition will occur with probability \(\lambda dt\)
- (we are dealing with a continuous-time Markov model)

Initial condition: \(p_1(0) = 1\)

\[
p_0 = 1 - e^{-\lambda t} \\
p_1 = e^{-\lambda t}
\]
**k-out-of-n Nonrepairable Systems**

\[
p'_n = -n\lambda p_n \\
p'_{n-1} = n\lambda p_n - (n - 1)\lambda p_{n-1} \\
\vdots \\
p'_k = (k + 1)\lambda p_{k+1} - k\lambda p_k \\
p_n + p_{n-1} + \ldots + p_k + p_F = 1
\]

Initial condition: \(p_n(0) = 1\)

\[
p_n = e^{-n\lambda t} \\
p_{n-1} = ne^{-(n-1)\lambda t}(1 - e^{-\lambda t}) \\
\vdots \\
p_k = \binom{n}{k}e^{-(n-k)\lambda t}(1 - e^{-\lambda t})^k \\
p_F = 1 - \sum_{j=k}^{n} p_j
\]

In this case, we do not need to resort to more general method of solving linear differential equations (LaPlace transform, to be introduced later)

The first equation is solvable directly, and each additional equation introduces only one new variable.
4.3 Modeling Repairable Systems

In steady state (equilibrium), transitions into/out-of each state must “balance out”

\[-\lambda p_1 + \mu p_0 = 0\]
\[p_1 + p_0 = 1\]

\[p_1 = \frac{\mu}{\lambda + \mu}\]
\[p_0 = \frac{\lambda}{\lambda + \mu}\]

Availability as a function of time:

\[A(t) = p_1(t) = \frac{\mu}{\lambda + \mu} + \frac{\lambda}{\lambda + \mu} e^{-(\lambda + \mu)t}\]

The label \(\mu\) on this transition means that over time \(dt\), repair will occur with probability \(\mu dt\) (constant repair rate as well as constant failure rate)

Steady-state availability

\[1\]
Multiple Failure States

In steady state (equilibrium), transitions into/out-of each state must “balance out”

\[-\lambda p_2 + \mu p_1 + \mu p_0 = 0\]
\[-\mu p_1 + \lambda_1 p_2 = 0\]
\[p_2 + p_1 + p_0 = 1\]

\[p_2 = \mu / (\lambda + \mu)\]
\[p_1 = \lambda_1 / (\lambda + \mu)\]
\[p_0 = \lambda_0 / (\lambda + \mu)\]

Safety evaluation:
Total risk of system is \(\sum_{\text{failure states}} c_j p_j\)
4.4 Modeling Fail-Soft Systems

\[-\lambda_2 p_2 + \mu_2 p_1 = 0\]
\[\lambda_1 p_1 - \mu_1 p_0 = 0\]
\[p_2 + p_1 + p_0 = 1\]

Let \(\delta = 1/[1 + \lambda_2/\mu_2 + \lambda_1\lambda_2/(\mu_1\mu_2)]\)

\[p_2 = \delta\]
\[p_1 = \delta\lambda_2/\mu_2\]
\[p_0 = \delta\lambda_1\lambda_2/(\mu_1\mu_2)\]

Performability evaluation:
Performability = \(\sum_{\text{operational states}} b_j p_j\)

Example: \(\lambda_2 = 2\lambda, \lambda_1 = \lambda, \mu_1 = \mu_2 = \mu\) (single repairperson or facility),
\(b_2 = 2, b_1 = 1, b_0 = 0\)

\[P = 2p_2 + p_1 = 2\delta + 2\delta\lambda/\mu = 2(1 + \lambda/\mu)/(1 + 2\lambda/\mu + 2\lambda^2/\mu^2)\]
Fail-Soft System with Imperfect Coverage

\[-\lambda_2 p_2 + \mu_2 p_1 = 0\]
\[\lambda_2 (1-c)p_2 + \lambda_1 p_1 - \mu_1 p_0 = 0\]
\[p_2 + p_1 + p_0 = 1\]

We solve this in the special case of \(\lambda_2 = 2\lambda, \lambda_1 = \lambda, \mu_2 = \mu_1 = \mu\)

Let \(\rho = \mu / \lambda\)

\[p_0 = 2[(1-c)\rho + 1]/[1 + (4 - 2c)\rho + 2\rho^2]\]
\[p_1 = 2\rho/[1 + (4 - 2c)\rho + 2\rho^2]\]
\[p_2 = \rho^2/[1 + (4 - 2c)\rho + 2\rho^2]\]

We can also consider coverage for the repair direction

If a unit’s malfunction goes undetected, the system fails.
4.5 Solving Markov Models

To solve linear differential equations with constant coefficients:
1. Convert to algebraic equations using LaPlace transform
2. Solve the algebraic equations
3. Use inverse LaPlace transform to find original solutions

\[ p'_1(t) = -\lambda p_1(t) + \mu p_0(t) \]
\[ p'_0(t) = -\mu p_0(t) + \lambda p_1(t) \]

LaPlace Transform Table

<table>
<thead>
<tr>
<th>Time domain</th>
<th>Xform domain</th>
</tr>
</thead>
<tbody>
<tr>
<td>( k )</td>
<td>( k/s )</td>
</tr>
<tr>
<td>( e^{-at} )</td>
<td>( 1/(s + a) )</td>
</tr>
<tr>
<td>( t^{n-1}e^{-at}/(n-1)! )</td>
<td>( 1/(s + a)^n )</td>
</tr>
<tr>
<td>( kh(t) )</td>
<td>( kH(s) )</td>
</tr>
<tr>
<td>( h(t) + g(t) )</td>
<td>( H(s) + G(s) )</td>
</tr>
<tr>
<td>( h'(t) )</td>
<td>( sH(s) - h(0) )</td>
</tr>
</tbody>
</table>

\[ sP_1(s) - p_1(0) = -\lambda P_1(s) + \mu P_0(s) \]
\[ sP_0(s) - p_0(0) = -\mu P_0(s) + \lambda P_1(s) \]

\[ P_1(s) = (s + \mu) / [s^2 + (\lambda + \mu)s] \]
\[ P_0(s) = \lambda / [s^2 + (\lambda + \mu)s] \]

\[ p_1(t) = \mu / (\lambda + \mu) + \lambda / (\lambda + \mu) e^{-(\lambda + \mu)t} \]
\[ p_0(t) = \lambda / (\lambda + \mu) - \lambda / (\lambda + \mu) e^{-(\lambda + \mu)t} \]
Inverse LaPlace Transform

\[ P_1(s) = \frac{s + \mu}{s^2 + (\lambda + \mu)s} \]
\[ P_0(s) = \frac{\lambda}{s^2 + (\lambda + \mu)s} \]

To find the solutions via inverse LaPlace transform:
1. Manipulate expressions into sum of terms, each of which takes one of the forms shown under \( H(s) \)
2. Find the inverse transform for each term

\[
\frac{s + \mu}{s^2 + (\lambda + \mu)s} = \\
\frac{1}{s + (\lambda + \mu)} + \frac{\mu}{s^2 + (\lambda + \mu)s}
\]
\[
\frac{1}{s^2 + (\lambda + \mu)s} = \frac{a}{s} + \frac{b}{s + (\lambda + \mu)}
\]
\[ 1 = a[s + (\lambda + \mu)] + bs \quad \rightarrow \quad a + b = 0 \]
\[ a = \frac{1}{\lambda + \mu} \quad \quad b = -\frac{1}{\lambda + \mu} \]

LaPlace Transform Table

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<tr>
<td>( h'(t) )</td>
<td>( sH(s) - h(0) )</td>
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4.6 Dependability Modeling in Practice

A birth-and-death process is a special case of Markov model with states appearing in a chain and transitions allowed only between adjacent states.

This model is used in queuing theory, where the customers’ arrival rate and provider’s service rate determine the queue size and waiting time.

Transition from state \( j \) to state \( j + 1 \) is an \textit{arrival} or \textit{birth}.

Transition from state \( j \) to state \( j - 1 \) is a \textit{departure} or \textit{death}.

Closed-form solution for state probabilities are difficult to obtain in general.

Steady-state prob.’s are easily obtained: \( p_j = p_0 \frac{\lambda_0 \lambda_1 \cdots \lambda_{j-1}}{(\mu_1 \mu_2 \cdots \mu_j)} \)
Birth-and-Death Process: Special Case 1

Constant *arrival* (birth) and *departure* (death) rates, infinite chain

Ex.: Bank customers arriving at random, and a single teller serving them
(State number is the customer queue size)

![Diagram](image)

Let \( r = \frac{\lambda}{\mu} \) be the ratio of birth and death rates

Steady-state prob.’s for the general case: \( p_j = p_0 \frac{\lambda_0 \lambda_1 \ldots \lambda_{j-1}}{(\mu_1 \mu_2 \ldots \mu_j)} \)

When \( \lambda_i = \lambda \) and \( \mu_i = \mu \), we have: \( p_j = p_0 (\frac{\lambda}{\mu})^j = p_0 \rho^j \)

\( p_0 (1 + \rho + \rho^2 + \ldots) = 1 \) yields \( p_0 = 1 - \rho \) and \( p_j = (1 - \rho)\rho^j \)

Finite chain: If \( n \) is the last state, then \( p_n = (1 - \rho)(\rho^n + \rho^{n+1} + \ldots) = \rho^n \)
Birth-and-Death Process: Special Case 2

Gracefully degrading system with \( n \) identical modules

State \( k \) corresponds to \( k \) modules being unavailable

If there are \( s \) identical service providers (repair persons), the departure or death transition rate is capped at \( s\mu \)

Steady-state probabilities for the \( n + 1 \) states with \( s \) service providers (M/M/s/n/n queue) can be found:

\[
p_j = (n - j + 1) (\lambda/\mu) p_{j-1}/j \quad \text{for } j = 1, 2, \ldots, s
\]
\[
p_j = (n - j + 1) (\lambda/\mu) p_{j-1}/s \quad \text{for } j = s + 1, s + 2, \ldots, n
\]

Equation for \( p_0 \) [Siew92], p. 347
TMR System with Repair

\[-3\lambda p_3 + \mu p_2 = 0\]
\[-(\mu + 2\lambda)p_2 + 3\lambda p_3 = 0\]
\[p_3 + p_2 + p_F = 1\]

Steady-state analysis of no use
\[p_3 = p_2 = 0, \quad p_F = 1\]

**Mean time to failure evaluation:**
See Textbook’s Example 4.11 for derivation
\[\text{MTTF} = \frac{5}{6\lambda} + \frac{\mu}{6\lambda^2} = \left[\frac{5}{6\lambda}\right](1 + 0.2\frac{\mu}{\lambda})\]

\[\text{MTTF Comparisons} \quad (\lambda = 10^{-6}/\text{hr}, \quad \mu = 0.1/\text{hr})\]
- Nonredundant \[\frac{1}{\lambda}\] \[1 \text{ M hr}\]
- TMR \[\frac{5}{6\lambda}\] \[0.833 \text{ M hr}\]
- TMR with repair \[\left[\frac{5}{6\lambda}\right](1 + 0.2\frac{\mu}{\lambda})\] \[16,668 \text{ M hr}\]

Assume the voter is perfect
Upon first module malfunction,
we switch to duplex operation
with comparison
The Dependability Modeling Process

1. Choose modeling approach
   - Combinational
   - State-space

2. Construct model

3. Derive model parameters

4. Solve model

5. Interpret results

6. Validate model and results

Iterate until results are satisfactory
Software Aids for Reliability Modeling

PTC Windchill (formerly Relex; specializes in reliability engineering)

Fault tree analysis: http://www.ptc.com/product/windchill/quality/fta
Markov analysis: http://www.ptc.com/product/windchill/quality/markov

University of Virginia
Galileo: http://www.cs.virginia.edu/~ftree

Iowa State University
HIMAP: http://ecpe.ece.iastate.edu/dcnl/Tools/tools_HIMAP.htm

See Appendix D, pp. 504-518, of [Shoo02] for more programs

More limited tools from MATLAB or some MATLAB-based systems