Memristor: The Fourth Fundamental Passive Circuit Element

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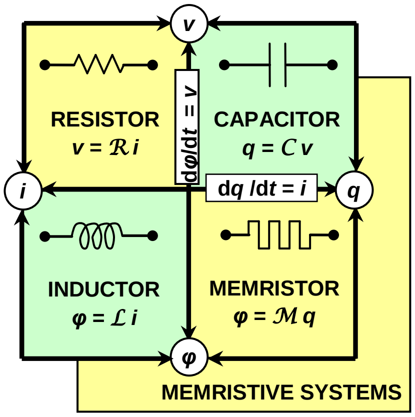
Abstract

The existence of a fourth passive circuit element was proposed by Chua in 1971 from fundamental symmetry arguments. Although he showed that such a 'memristor' had many interesting and useful circuit properties, until now no one has presented a physical model or example of such a device and the idea has almost been forgotten. We show here using a simple analytical example that memristance arises naturally in two-terminal devices for which electronic and atomic transport are coupled. These results serve as the foundation for understanding a wide range of hysteretic current-voltage behavior observed in next-generation non-volatile resistive RAM devices, including nanoscale titanium oxide crosspoint switches built in our laboratory.

1. Problem Statement

Anyone who ever took an electronics laboratory class is familiar with the basic passive circuit elements: resistor $\mathcal{R}$, capacitor $\mathcal{C}$, and inductor $\mathcal{L}$. However, using basic symmetry arguments in 1971 Leon Chua proposed that there should be a fourth passive two-terminal element, which he called a memristor $\mathcal{M}$ (an acronym for memory and resistor).[1] He noted that there should be six different mathematical relations among the four fundamental circuit variables: current i, voltage v, charge q, and flux φ. Since two of these relations are determined from definitions of the variables, there must be four basic circuit elements described by the remaining relations (Fig. 1). The 'missing element' that provides a functional relation between charge and flux, $\varphi = \mathcal{M}\, q$, is exactly what defines a memristor. In the trivial case in which $\mathcal{M}$ is a constant, a memristor is identical to a resistor, and thus of no special interest. However, if $\mathcal{M}$ is itself a function of q to yield a nonlinear circuit element, then things become quite interesting. Chua showed that in general the i-v characteristic of such a nonlinear relation between q and φ for a sinusoidal input was a frequency-dependent Lissajou figure, and that no combination of nonlinear $\mathcal{R}$, $\mathcal{C}$ and $\mathcal{L}$ components could duplicate the circuit properties of $\mathcal{M}$ (although including active circuit elements such as amplifiers could do so). Since most valuable circuit functions come from nonlinear characteristics and $\mathcal{M}$ has properties that require extensive active circuitry to reproduce, integrated-circuit compatible memristors could significantly improve the performance of electronics without increasing transistor density. However, until now there has been neither a physical model nor a material example of $\mathcal{M}$.



Memristor:

$$v = R(w)i \qquad (1a)$$

$$\frac{dw}{dt} = i \qquad (1b)$$

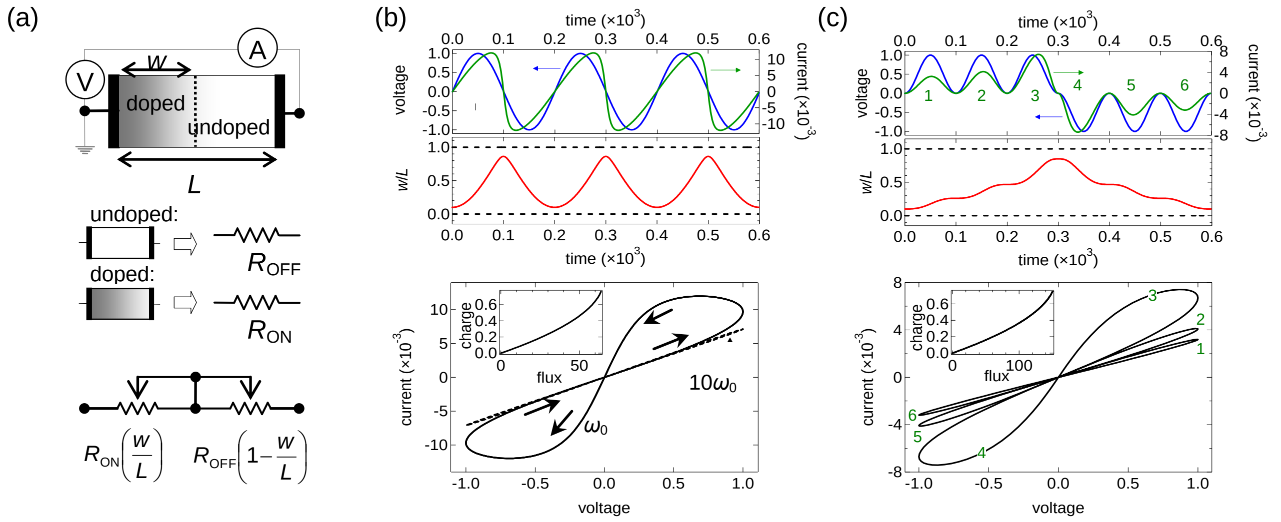
Memristive system:

$$v = R(w,v)i \qquad (2a)$$

$$\frac{dw}{dt} = f(w,i) \qquad (2b)$$

Figure 1. The four fundamental two-terminal circuit elements. The resistor and memristor are subsets of the more general class of dynamical memristive systems.

The most useful mathematical definition of a current-controlled memristor for circuit analysis is described by equation 1, where w is the *state variable* of the device and R is a generalized resistance that depends upon the internal state of the device. In 1976, Chua and Kang generalized the memristor concept to a much broader class of nonlinear dynamical systems they called *memristive systems*,[1] described by the equations 2, where w can be a vector while R and f may be functions of time in the general case. Equation 2a distinguishes a memristive device from an arbitrary dynamical one, namely that no current flows through it when the voltage drop across it is zero. Chua and Kang were able to show that some devices and systems, notably thermistors, Josephson



$$v(t) = \left[R_{\mathrm{ON}} \frac{w(t)}{L} + R_{\mathrm{OFF}} \left(1 - \frac{w(t)}{L} \right) \right] i(t) \quad \ldots(3a)$$

$$\frac{dw(t)}{dt} = \mu_{\mathrm{V}} \frac{R_{\mathrm{ON}}}{L} i(t) \quad \ldots(3b)$$

$$w(t) = \mu_{\mathrm{V}} \frac{R_{\mathrm{ON}}}{L} q(t) \quad \ldots(4)$$

Figure 2. A two-terminal resistive switch: (a) a schematic diagram with a simplified equivalent circuit, and (bottom) equations of motion for the state variable *w(t)* and resulting *i*-*v* relation analagous to equations 2a, 2b. (b) and (c) calculated memristor behavior under applied voltages $v_0\sin(\omega_0 t)$ and $\pm v_0\sin^2(\omega_0 t)$, and resistance ratios $R_{\mathrm{OFF}}/R_{\mathrm{ON}} = 160$ and 380, correspondingly. In all figures axes are dimensionless with voltage, current, time, flux, and charge, and are conveniently drawn in units of $v_0 = 1$ V, $i_0 \equiv v_0/R_{\mathrm{ON}} = 10$ mA, $t_0 \equiv 2\pi/\omega_0 \equiv L^2/\mu_{\mathrm{V}} v_0 = 10$ ms, $v_0 t_0$, and $i_0 t_0$, respectively.

junctions, neon bulbs, and even the Hodgkin-Huxley model of a neuron, could be modeled with memristive equations. However, there was still no connection between the mathematics and any known physical properties.

2. Our Approach: theory and comparison with experiment

Electrical switching in thin film devices has recently attracted significant attention since such a technology may enable scaling of logic and memory circuits well beyond CMOS limits.[2] Cases of hysteretic switching have indeed been reported in the literature for nearly 50 years,[3] yet to this day remain poorly explained. While the microscopic nature of resistance switching and charge-transport in such devices are still under substantial debate, one concensus is that the hysteresis requires some kind of atomic rearrangement that modulates the electronic current. Based on this assumption, we consider a simple but insightful model in which a thin semiconductor film of thickness L is sandwiched between two metal contacts, as shown in Fig. 2a. The total resistance of the device is determined by two variable resistors connected in series (Fig. 2a), where the resistances are given for the full length L of the device. More specifically, the device has one region with a high concentration of dopants (positive ions or vacancies) having low resistance R_{ON}, and the remainder with much higher resistance R_{OFF} that has a low (or essentially zero) dopant concentration. The application of an external bias $v(t)$ across the device will move the boundary between the two regions by causing the charged dopants to drift. For the case of ohmic electronic conduction and linear ionic drift in a uniform field with average mobility μ_{v} we can write equation 3, which yields a simple formula for $w(t)$. The two coupled equations of motion (Eq. 3a,b) for the dopants and the electrons in this system satisfy the normal form for a current (or charge) controlled memristor (Eq 1a,b). The significant issue here that was not anticipated by Chua is that the state variable w, which in this case specifies the distribution of dopants in the device, is bounded between 0 and L. The state variable is proportional to the charge until w approaches L, e.g. the condition of 'hard' switching (large voltage excursions or long times under bias). As long as the system remains in the memristor regime, any symmetrical ac voltage bias results in single loop *i*-*v* hysteresis that collapses to a straight line for high frequencies (Fig. 2b). Multiple continuous states can be also obtained with any asymmetry in the applied bias (Fig. 2c). Obviously, Eq. 4 is not valid if w is forced outside the interval $[0, L]$. Different hard switching cases are defined by imposing a variety of boundary conditions, such as assuming that once w hits either of the boundaries it remains constant until the voltage reverses polarity. In such a case, the device satisfies the normal form for a current controlled memristive system (Eq. 2a,b).

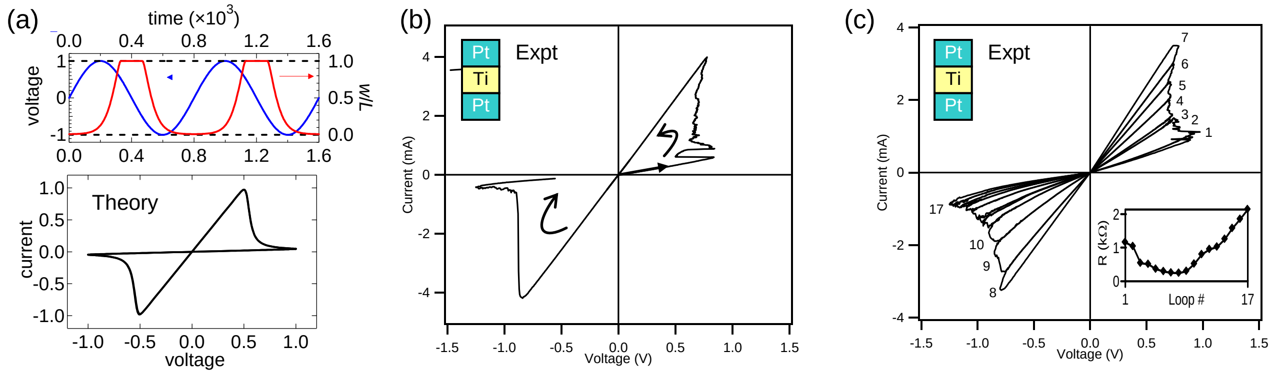


Figure 3. A memristive switch: (a) simulation results and (b) experimental *i-v* curve for a 5 nm thick TiOx crosspoint junction device driven on-off-on, and (c) the same device driven with a two-part single polarity pulse train analagous to Fig. 2(c) above.

A memristive system also arises any time the ionic equation of motion is nonlinear, which is likely when a few volts applied over a few nanometers yield enormous fields. Figure 3a shows such a case when the drift is nonlinear for w close to 0 or L. When the boundary is approached very closely, the switching event requires a significantly larger amount of charge (or even a threshold voltage) than for linear drift. Therefore, the switching is essentially binary because the ON and OFF states can be held much longer if the voltage does not exceed a specific threshold.

The model of Eq. 3a,b mimics well many features that have been described as bipolar switching, i.e. when voltages of opposite polarity are required for switching a device ON and OFF. This type of behavior has been experimentally observed in various material systems: organic films; chalcogenides where switching was attributed to ion migration rather than a phase transition; and metal oxides – for review see, e.g., Refs. 2, 4. For example, both multi-state and binary switching that are similar to those modeled in Figs. 2c and 3a, respectively, have been observed, with some showing dynamical negative differential resistance.

We have built and tested many devices such as those shown in Fig. 2a, for which the semiconductor was TiO_2 and the dopant was oxygen vacancies. An *i-v* curve for one of these devices is shown in Fig. 3b, and can be seen to be qualitatively similar to the simple physical model. In order to provide a better agreement between the model and the experiment, a more accurate representation of both the electronic transport mechanism and the charged vacancy drift are required; these are currently under investigation in our lab. The TiO_2 material is compatible with standard silicon processing, and in fact is closely related to the new gate oxide (HfO_2) used in 45 nm CMOS. Thus, we anticipate that we will be able to integrate memristors and memristive systems with standard CMOS, and in fact a test chip with such devices is presently in fabrication by the Technology Development Organization in Corvallis.

3. Next Steps: applications

The rich hysteretic *i-v* behavior observed in many thin film devices can now be understood as a memristive system defined by two coupled equations of motion: one for atomic degrees of freedom that defines the internal state of the device and another for the electronic transport. This behavior has become more relevant now that the thin films in many devices are just a few nanometers thick, so that even a low voltage bias corresponds to a large field and can cause charged dopants to drift. This could be a major liability, but by controlling the materials it is now becoming possible to include memristors and memristive systems in integrated circuits. This has the potential for dramatically extending the functionality and efficiency of electronics, as long as the dynamical nature of the devices is understood and properly utilized. Examples of such circuits would be semi-non-volatile memories, nano-scale latches, new types of ultra-dense nano-logic gates, and learning networks that require a synapse-like function.

References

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