Comprehensive Compact Model of $I$-$V$ Characteristics, Temperature Dependency, Variations, and Noise for Integrated Reproducible Metal-Oxide Memristors

H. Nili, M. Prezioso, and D. Strukov

UC Santa Barbara, Santa Barbara, CA 93106-9560, U.S.A. email: \{hnili, mprezioso, strukov\}@ece.ucsb.edu

Abstract—We present a comprehensive empirical model for the crossbar integrated metal-oxide memristors. The model consists of static and dynamic equations, which are obtained by fitting a large amount of experimental data, collected on several hundred devices. The static equation describes the device current, at non-disturbing memory state stress conditions, as a sum of three components, representing the average behavior and its device-to-device and temporal variations. All three components are expressed as functions of the applied voltage, ambient temperature, and the current memory state. The dynamic equation models the change in the memory state as a function of the amplitude and duration of the applied voltage pulse and the current memory state, and is also described as a sum of the average and device-to-device variation terms. Both equations are explicit, computationally inexpensive, and suitable for SPICE modeling. At the same time, the model has good predictive power, which is supported by the validation results. We expect that the presented model will be very useful for realistic simulations of various memristor-based circuits, such as, e.g., in-situ trained mixed-signal neuromorphic networks, which rely on analog properties of the memory devices and are highly sensitive to their non-ideal behavior. Although our results are obtained for particular, though representative devices, the modelling approach is quite general and can be applied to other nonvolatile memory technologies.

I. INTRODUCTION

Though there has been significant progress in understanding the physics of operation for the metal-oxide ReRAM (also called memristors) [1], the development of accurate and comprehensive compact models is proven to be still very challenging. For the devices based on strongly-correlated materials, this is in part due to very rich physics so that multiple mechanisms, e.g., switching due to ferroelectric domains, metal insulator transition, and drift of the defects, can be involved in the device operation [1]. For amorphous material devices, the modeling is in part challenging due to reproducibility and variation issues.

Indeed, the majority of the reported work is on physics-based models [2-4], which are very useful for verifying physical mechanisms, and guiding towards engineering of better devices. However, such models are generally inadequate for accurate and fast simulations, in part because of their focus on just certain aspects of the device operation, but also due to their implicit form, such as system of coupled differential equations. The reported compact and SPICE models [5-8] are not sufficiently detailed (e.g., lacking device variation) and accurate, largely because they are derived based on the simplified assumptions for the resistive switching and electron transport mechanisms.

The need for accurate and comprehensive compact model of memristors is, however, very acute now, given the recent advances in this technology and the increased focus on investigating memristor’s potentials in various applications. The main contribution of this paper is the development of such model using an empirical approach. The presented model is a substantial improvement over our previous work [9], which neglected non-ideal behavior, time and temperature dependence, and was developed based on the data from a single discrete device.

II. INTEGRATED METAL-OXIDE MEMRISTORS

The model was developed for Pt/Al$_2$O$_3$/TiO$_{2-x}$/Ti/Pt devices, integrated in the 20×20 crossbar arrays, with 200-nm lines separated by 400-nm gaps (Fig. 1). The fabrication technique, which is suitable for 3D CMOS integration [10], as well as extensive endurance and retention data, have been reported in Refs. [10, 11]. Similarly to our prior results [10, 11], the studied crossbar devices have fairly uniform $I$-$V$ characteristics, with a relatively narrow spread of the set and reset voltages (Fig. 1c) - a critical requirement for the model development.

III. GENERAL MODELING APPROACH

The general idea of our approach is to model the device behavior with two equations: $I = S(G_0,V,T)$ and $\Delta G_0 = D(G_0,V,p,t_p)$. The first, “static” equation describes device current ($I$) as a function of the applied voltage ($V$), ambient temperature ($T$), and current memory state ($G_0 \equiv I(0.1V)/0.1$ V), at relatively small ($\sim 0.4$ V in our case) voltage biases that do not modify the state of the device within the studied temperature range. The second, “dynamic” equation describes the change in the device’s memory state as a function of its initial state after the application of the voltage pulse with amplitude $V_p$ and duration $t_p$. Such method of separating static and dynamic behavior works well for the most practical devices with strongly-nonlinear switching kinetics.

Specifically, these two equations are expressed as $S = S_m(G_0,V,T) + \delta S(G_0,V,T) + S_t(V)$ and $D = D_m(G_0,V,p,t_p) + \delta D(G_0,V,p,t_p)$, where $S_m$ and $D_m$ represent, the device’s static noiseless expected $I$-$V$ and the expected $\Delta G$, respectively, while $\delta S$ and $\delta D$ are their normally-distributed stochastic device-to-device variations, and $S_t(V)$ is temporal variation due to device’s intrinsic noise.
IV. MODELLING RESULTS

To model the static equation, we recorded the $I$-$V$ curves in the range -0.4 V to +0.4 V from 324 devices at 6 different temperatures: RT, 40, 55, 70, 85 and 100 °C. All the curves have been then fitted with a cubic polynomial of the form $A_1(G_0, T)V + A_3(G_0, T)V^3$ mainly due to its simplicity. Two functions (of $G_0$ and $T$) are then fitted to extracted $A_1$ and $A_3$ parameters to model their averages (Fig. 2a, b). $\delta S$ was obtained in the same cubical polynomial form by finding coefficient of variations (CVs) for $A_1$ and $A_3$ (Figs. 2c, d, 7). To make the model more representative, 22 outlier devices for which parameter $A_1$ deviated by more than 50% from a quadratic surface fitting were not considered (Fig. 1d). Fig. 2e shows the $R^2$ value between all the experimental $I$-$V$'s and the ones defined by $S_m$ model. The plot confirms sufficient goodness-of-fitting in the $10^{-5}$ S to $2 \times 10^{-4}$ S range. The static model accuracy is cruder for higher temperatures and lower conductances. The simple noise model was derived from the measured data (Fig. 3), which are comparable with previous work [12].

The same 324 devices were used for deriving the dynamic model. In this case, all devices were first randomly initialized to memory states within 3.5 μS to 300 μS range. Each device was then subjected to 6,000 voltage pulses with random polarity and amplitude (from the ranges [-0.8 V, -1.5 V] / [0.8 V to 1.15 V] for set / reset) and duration (from 100 ns to 100 ms), resulting in a total dataset size of ~2 million points that is covering uniformly the whole range of memory states (Fig. 4a). To avoid shunting and irreversible damage, the devices were not stressed beyond the range of studied conductances.

Functions $D_m$ and $\delta D$ were found separately for 8 ranges of memory states. For each range, $D_m$ was found by fitting tanh functions to the averages of the data across all the devices and specific memory state range (Fig. 4a, b). The CVs for the averaged data, representing device-to-device variations, were then fitted to obtain $\delta D$ using two third-order polynomials with four parameters (Fig. 4c, d). Note that fitting parameters for $D_m$ and $\delta D$ are all functions of $G_0$ – see, e.g., Figs. 4b and 7 for specific cases. The dynamic model was also validated on a set of device data, which were not used in the fitting (Fig. 5). The validation results are consistent with the model.

The functional forms and all fitting parameters are summarized in Fig. 7.

V. DISCUSSION AND SUMMARY

One of the important tradeoffs in the model, which is an important future work, is the complexity of the fitted functions versus the accuracy of the model. The choice of these functions can be guided by plausible physical mechanisms of the device operation. In fact, the polynomial function used in static equation fitting is already motivated by plausible conduction model, e.g., with the linear part representing an ohmic characteristics of vacancy-doped conductive filaments, while the non-linear one approximating the bulk trap-limited conduction of the insulating parts of the device and/or the interface injection. Not surprisingly, as Fig. 2b shows, the parameter $A_3$ gets smaller as the state of the device gets more conductive and as the temperature rises, which is explained by more dominant ohmic conduction.

The dynamic model can be further simplified by observing the experimental data for the change in the memory state as a function of the power, approximated as $V_p^2(G_0+\Delta G_0)/2$ and $\delta_0$. As Fig. 6 shows, the change in state is roughly exponentially proportional to the switching power, which is expected from thermally activated resistive switching mechanism. (Interestingly, extrapolation of this data to larger powers shows a possibility for sub-ns switching.)

In summary, our paper presents a comprehensive compact model for metal-oxide memristors, which is developed via empirical fitting of the measured data. The model includes static and dynamic characteristics, their device-to-device variations, temperature dependence and noise.

ACKNOWLEDGMENT

This work was supported by DARPA’s UPSIDE program (HR0011-13-C-0051 UPSIDE) via BAE Systems, NSF grant CCF 1740352, and DENSO CORP., Japan. The authors are grateful B. Chakrabarti, I. Kataeva, F. Merrikh Bayat, and K. K. Likharev for useful discussions and technical support.

REFERENCES

Fig. 1. Reproducible integrated metal-oxide memristors: (a) Top view SEM image of a 20 × 20 bilayer metal-oxide crossbar. (b) Cross-sectional TEM view of a single device in the crossbar array with different metal and oxide layers identified. (c) Representative I-V characteristics and set and reset threshold statistics. (d) Conductance state retention (at 0.1 V), for the crossbar devices after obtaining all I-V data for static modelling at small non-disturbing biases (< 0.4 V) and ambient temperatures up to 100 °C. 'x's denote the devices removed from modelling due to abnormal behavior.

Fig. 2. Static model: (a, b) data (dots) and fitting surfaces for the model parameters $A_f(G_m, T)$, $A_r(G_m, T)$ and (c, d) their corresponding coefficient of variations (CVs), i.e. the standard deviation normalized over the average. (The data has been binned over conductance values.) Panel (e) shows the goodness-of-fitting through the $R^2$ measure obtained with the final $S_0$ function applied to all the experimental I-V curves.

Fig. 3. Noise model: (a) Normalized power spectral density ($S_{f/V}^2$) measured at reading voltage of 0.3 V. Spectra have been collected on all 324 devices, binned in 6 conductance ranges and averaged. Note that the corner frequency is lower for the lower the conductance state because of the noise floor of the measurement setup. (b) C coefficient of variation of the current time series data collected from all the devices at 0.1 V, 0.2 V and 0.3 V, shown as a function of $G_0$. Data are collected for 1 s at a sampling rate of 200 kHZ.
Fig. 4. Dynamic model: (a) Measured absolute change in memory state ($\Delta G_M$) as a function of the amplitude ($V_0$) and duration ($t_0$) of the applied voltage pulse for several ranges of memory states. On each panel, dots show the measured data for the device with the initial memory states in the particular range. The surfaces show the corresponding fitted functions. (b) Fitting parameters for $\Delta G_M$ function of the dynamic model. (c, d) Coefficient of variation ($\sigma/G_M$) and its corresponding polynomial fit for a specific representative conductance range.

Fig. 5. Dynamic model validation: Lognormal probability plots of the absolute normalized difference between measured values of the change in memory state and those predicted by the dynamic model $\Delta G_M$ for 10 devices, which were not used in the fitting. Each set of data shown with the same color is for a particular pulse duration and all voltage amplitudes.

Fig. 6. Measured absolute change in conduction (at 0.1 V) shown as a function of the applied power for (a, b) set and (c, d) rest transitions for two shown values of the voltage pulse duration.

**Stiff Model**

<table>
<thead>
<tr>
<th>$S_{\text{if}}(V, T)$</th>
<th>$A_1(G_0, T) V + A_2(G_0, T) V^2$</th>
<th>$A_3(G_0, T) V^3 + A_4(G_0, T) V^4 + A_5(G_0, T) V^5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S_{\text{if}}(V, T)$</td>
<td>$A_1(G_0, T) V + A_2(G_0, T) V^2$</td>
<td>$A_3(G_0, T) V^3 + A_4(G_0, T) V^4 + A_5(G_0, T) V^5$</td>
</tr>
<tr>
<td>$A_1(G_0, T)$</td>
<td>$A_1(G_0, T) V + A_2(G_0, T) V^2$</td>
<td>$A_3(G_0, T) V^3 + A_4(G_0, T) V^4 + A_5(G_0, T) V^5$</td>
</tr>
<tr>
<td>$A_2(G_0, T)$</td>
<td>$A_1(G_0, T) V + A_2(G_0, T) V^2$</td>
<td>$A_3(G_0, T) V^3 + A_4(G_0, T) V^4 + A_5(G_0, T) V^5$</td>
</tr>
<tr>
<td>$A_3(G_0, T)$</td>
<td>$A_1(G_0, T) V + A_2(G_0, T) V^2$</td>
<td>$A_3(G_0, T) V^3 + A_4(G_0, T) V^4 + A_5(G_0, T) V^5$</td>
</tr>
<tr>
<td>$A_4(G_0, T)$</td>
<td>$A_1(G_0, T) V + A_2(G_0, T) V^2$</td>
<td>$A_3(G_0, T) V^3 + A_4(G_0, T) V^4 + A_5(G_0, T) V^5$</td>
</tr>
<tr>
<td>$A_5(G_0, T)$</td>
<td>$A_1(G_0, T) V + A_2(G_0, T) V^2$</td>
<td>$A_3(G_0, T) V^3 + A_4(G_0, T) V^4 + A_5(G_0, T) V^5$</td>
</tr>
<tr>
<td>$S_{\text{if}}(V, T)$</td>
<td>$A_1(G_0, T) V + A_2(G_0, T) V^2$</td>
<td>$A_3(G_0, T) V^3 + A_4(G_0, T) V^4 + A_5(G_0, T) V^5$</td>
</tr>
</tbody>
</table>

**Dynamic Model**

$\Delta G_M(V)$

$$\begin{align*}
\Delta G_M(V) &= c_0 \left[ 1 - \tanh \left( \frac{c_0 \left( \log(V) - t_0^{\text{ref}} \right)}{c_0} \right) \right] \left( \tanh \left( a_0 \left( G_D - G_M \right) \right) - \tanh \left( b_0 \left( G_M - G_D \right) \right) \right) + \frac{C}{1 + \left( \frac{V}{V_0} \right)^{c_0}} \\
&\quad - c_0 \left[ 1 - \tanh \left( \frac{c_0 \left( \log(V) - t_0^{\text{ref}} \right)}{c_0} \right) \right] \left( \tanh \left( a_0 \left( G_D - G_M \right) \right) - \tanh \left( b_0 \left( G_M - G_D \right) \right) \right) + \frac{C}{1 + \left( \frac{V}{V_0} \right)^{c_0}} \\
&\quad + \Delta G_0(V) + \Delta G_0(V) \log(V) + \Delta G_0(V) \log(V) + \Delta G_0(V) \log(V) + \Delta G_0(V) \log(V) + \Delta G_0(V) \log(V) + \Delta G_0(V) \log(V)
\end{align*}$$

**Model Parameters**

<table>
<thead>
<tr>
<th>Model Parameters</th>
<th>$G_M$</th>
<th>$G_D$</th>
<th>$V_0$</th>
<th>$t_0$</th>
<th>$C$</th>
<th>$C_0$</th>
<th>$C_0$</th>
<th>$C_0$</th>
<th>$C_0$</th>
<th>$C_0$</th>
<th>$C_0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$G_M$</td>
<td>5.60 - 10.0 μS</td>
<td>5.60 - 10.0 μS</td>
<td>5.60 - 10.0 μS</td>
<td>5.60 - 10.0 μS</td>
<td>5.60 - 10.0 μS</td>
<td>5.60 - 10.0 μS</td>
<td>5.60 - 10.0 μS</td>
<td>5.60 - 10.0 μS</td>
<td>5.60 - 10.0 μS</td>
<td>5.60 - 10.0 μS</td>
<td>5.60 - 10.0 μS</td>
</tr>
<tr>
<td>$G_D$</td>
<td>5.60 - 10.0 μS</td>
<td>5.60 - 10.0 μS</td>
<td>5.60 - 10.0 μS</td>
<td>5.60 - 10.0 μS</td>
<td>5.60 - 10.0 μS</td>
<td>5.60 - 10.0 μS</td>
<td>5.60 - 10.0 μS</td>
<td>5.60 - 10.0 μS</td>
<td>5.60 - 10.0 μS</td>
<td>5.60 - 10.0 μS</td>
<td>5.60 - 10.0 μS</td>
</tr>
<tr>
<td>$V_0$</td>
<td>5.60 - 10.0 μS</td>
<td>5.60 - 10.0 μS</td>
<td>5.60 - 10.0 μS</td>
<td>5.60 - 10.0 μS</td>
<td>5.60 - 10.0 μS</td>
<td>5.60 - 10.0 μS</td>
<td>5.60 - 10.0 μS</td>
<td>5.60 - 10.0 μS</td>
<td>5.60 - 10.0 μS</td>
<td>5.60 - 10.0 μS</td>
<td>5.60 - 10.0 μS</td>
</tr>
<tr>
<td>$t_0$</td>
<td>5.60 - 10.0 μS</td>
<td>5.60 - 10.0 μS</td>
<td>5.60 - 10.0 μS</td>
<td>5.60 - 10.0 μS</td>
<td>5.60 - 10.0 μS</td>
<td>5.60 - 10.0 μS</td>
<td>5.60 - 10.0 μS</td>
<td>5.60 - 10.0 μS</td>
<td>5.60 - 10.0 μS</td>
<td>5.60 - 10.0 μS</td>
<td>5.60 - 10.0 μS</td>
</tr>
<tr>
<td>$C$</td>
<td>5.60 - 10.0 μS</td>
<td>5.60 - 10.0 μS</td>
<td>5.60 - 10.0 μS</td>
<td>5.60 - 10.0 μS</td>
<td>5.60 - 10.0 μS</td>
<td>5.60 - 10.0 μS</td>
<td>5.60 - 10.0 μS</td>
<td>5.60 - 10.0 μS</td>
<td>5.60 - 10.0 μS</td>
<td>5.60 - 10.0 μS</td>
<td>5.60 - 10.0 μS</td>
</tr>
<tr>
<td>$C_0$</td>
<td>5.60 - 10.0 μS</td>
<td>5.60 - 10.0 μS</td>
<td>5.60 - 10.0 μS</td>
<td>5.60 - 10.0 μS</td>
<td>5.60 - 10.0 μS</td>
<td>5.60 - 10.0 μS</td>
<td>5.60 - 10.0 μS</td>
<td>5.60 - 10.0 μS</td>
<td>5.60 - 10.0 μS</td>
<td>5.60 - 10.0 μS</td>
<td>5.60 - 10.0 μS</td>
</tr>
<tr>
<td>$C_0$</td>
<td>5.60 - 10.0 μS</td>
<td>5.60 - 10.0 μS</td>
<td>5.60 - 10.0 μS</td>
<td>5.60 - 10.0 μS</td>
<td>5.60 - 10.0 μS</td>
<td>5.60 - 10.0 μS</td>
<td>5.60 - 10.0 μS</td>
<td>5.60 - 10.0 μS</td>
<td>5.60 - 10.0 μS</td>
<td>5.60 - 10.0 μS</td>
<td>5.60 - 10.0 μS</td>
</tr>
<tr>
<td>$C_0$</td>
<td>5.60 - 10.0 μS</td>
<td>5.60 - 10.0 μS</td>
<td>5.60 - 10.0 μS</td>
<td>5.60 - 10.0 μS</td>
<td>5.60 - 10.0 μS</td>
<td>5.60 - 10.0 μS</td>
<td>5.60 - 10.0 μS</td>
<td>5.60 - 10.0 μS</td>
<td>5.60 - 10.0 μS</td>
<td>5.60 - 10.0 μS</td>
<td>5.60 - 10.0 μS</td>
</tr>
<tr>
<td>$C_0$</td>
<td>5.60 - 10.0 μS</td>
<td>5.60 - 10.0 μS</td>
<td>5.60 - 10.0 μS</td>
<td>5.60 - 10.0 μS</td>
<td>5.60 - 10.0 μS</td>
<td>5.60 - 10.0 μS</td>
<td>5.60 - 10.0 μS</td>
<td>5.60 - 10.0 μS</td>
<td>5.60 - 10.0 μS</td>
<td>5.60 - 10.0 μS</td>
<td>5.60 - 10.0 μS</td>
</tr>
</tbody>
</table>

Fig. 7. Summary of the model and its parameters.