DISTRIBUTED BEAMFORMING USING MOBILE ROBOTS

Arjun Muralidharan and Yasamin Mostofi

Department of Electrical and Computer Engineering, University of California Santa Barbara, USA

ABSTRACT

We consider the case where a team of unmanned vehicles are tasked with distributed beamforming in order to cooperatively transmit a message to a remote station. We propose a joint motion and communication optimization framework where the robots move in order to find locations that satisfy the given reception quality requirement while minimizing the overall motion energy consumption. For the case where the channel is perfectly known, we show that this problem can be posed as a knapsack problem and show the underlying trends of the optimum solution. We then extend our approach to the case where the channel is not known over the space. We show how the previously proposed channel prediction framework can be integrated with path planning for distributed robotic beamforming under motion energy constraints. Finally, we present extensive simulation results in realistic communication environments.

***Index Terms*—** Mobile robots, distributed beamforming, multiple-choice knapsack problem.

1. INTRODUCTION

A wireless sensor network is a spatially distributed network of sensor nodes with several potential applications [1]. Recent advances in robotics have further enabled the possibility of a network of unmanned vehicles. With the addition of mobility, the network can have several potential applications such as search and rescue, emergency response, or surveillance.

Relation to past work: In this paper, we consider a network of unmanned vehicles. In such networks, interesting interplay between communication and motion planning arises, which has opened up new multidisciplinary areas such as communication-aware path planning [2, 3, 4]. For instance, a node can utilize its mobility to find a spot better for communication [5] or to form a communication relay network [6]. In [7], robots exploit their mobility to move into better positions in order to act as collaborative relay beamformers. In the communication literature, cooperative communication strategies such as distributed beamforming have been studied and utilized in the context of fixed nodes [8]. Such strategies take advantage of transmit diversity and communicate the same message, with optimum weights, from multiple nodes. In this paper, we consider distributed cooperative beamforming with unmanned vehicles, in order to enable successful communication with a remote station. The mobile nodes all have the message that needs to be communicated to a remote node. Each node can incur motion energy to move to a better place for cooperative beamforming. We are then interested in finding the optimum final positions of the nodes under reception quality constraints. There are two main underlying challenges. First, each node needs to have an assessment of the link quality when communicating from an unvisited location over the field. In order to enable this, we utilize our previously-proposed probabilistic channel prediction framework [9], which allows each node to assess the channel quality at an unvisited location, based on a small number of a priori channel samples. Introducing cooperative beamforming will further result in new challenges for path planning as each node can not decide on its own final destination without considering other nodes. We then pose and analyze the overall joint motion and communication optimization problem. In Section 3, we start with analyzing the case where the channel is perfectly estimated by the nodes, in order to gain insights into the optimum solution and the underlying trends. We show that the minimum motion energy cooperative beamforming problem can be posed as a multiple-choice knapsack problem [10], which can be solved efficiently with existing tools. We further show underlying trends of the optimum solution, e.g., how different channel parameters will impact the total motion energy consumption. In Section 4, we then extend our framework to the case where the nodes do not know the channel over the field and have to probabilistically assess it. Our results show the impact of channel assessment uncertainty on the optimum solution.

2. PROBLEM SETUP

In this section we start by introducing our utilized motion model. We then summarize how each robot can probabilistically and realistically assess the channel at unvisited locations over the workspace, which is key for its path planning.

2.1. Motion Energy Modeling

In this paper, we take the motion energy to be proportional to the distance traveled, i.e. Motion Energy $= \kappa_M d$ where d is the distance traveled and κ_M is a constant that depends on factors such as friction, terrain type, and the mass of the vehicle. As several studies in the robotics literature indicate, this is a good model for wheeled robots [11, 3, 12].

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2.2. Communication Modeling

In order for each robot to move to a proper spot for cooperative beamforming, it needs an assessment of the communication quality at any position in the workspace.[1] Realistically, however, the robots would only have a few samples of the channel collected along their trajectories or from prior operations in the same workspace. Then, each robot needs to predict the channel at unvisited locations, based on these a priori channel samples. In this section, we briefly summarize our previous work on probabilistically predicting the spatial variations of wireless channels, enabling the robots to realistically assess the channel over the field.

2.2.1. *Overview of Probabilistic Channel Modeling [13]*

A communication channel is best modeled as a multi-scale random process with three major dynamics: path loss, shadowing and multipath fading [13]. Let $\Gamma(x)$ denote the received channel power from a transmitter at location $x \in \mathcal{W}$ ($\mathcal{W} \subseteq \mathbb{R}^2$ is the workspace) to the remote station (located at x_b). The received channel power in the dB domain, $\Gamma_{\text{dB}}(x) = 10\log_{10}(\Gamma(x))$, can be expressed as $\Gamma_{\text{dB}}(x) = \Gamma_{\text{PL,dB}}(x) + \Gamma_{\text{SH,dB}}(x) + \Gamma_{\text{MP,dB}}(x)$ where $\Gamma_{\text{SH,dB}}$ and $\Gamma_{\text{MP,dB}}$ are random variables denoting the impact of shadowing and multipath respectively, and $\Gamma_{\text{PL,dB}}(x) = K_{\text{dB}} - 10n_{\text{PL}}\|x - x_b\|$ is the distance-dependent path loss with n_{PL} representing the path loss exponent. $\Gamma_{\text{SH,dB}}(x)$ is best modeled as a Gaussian random variable with an exponential spatial correlation: $\mathbb{E}\left\{\Gamma_{\text{SH,dB}}(x_1)\Gamma_{\text{SH,dB}}(x_2)\right\} = \alpha e^{-\|x_1 - x_2\|/\beta}$ where α is the shadowing power and β is the decorrelation distance.

2.2.2. *Overview of Realistic Channel Prediction [9],[4]*

Let $\theta = [K_{\text{dB}}\ n_{\text{PL}}]^{\text{T}}$ denote the vector of path loss parameters. Let Y represent the stacked vector of m a priori-gathered received channel power measurements (in dB) that are collected in the same environment, and $Q = \{q_1, \cdots, q_m\}$ denote the corresponding positions. Then, a Gaussian random variable, $\Gamma_{\text{dB}}(q)$, with the mean of $\overline{\Gamma}_{\text{dB}}(q) = \mathbb{E}\{\Gamma_{\text{dB}}(q) \mid Y, \hat{\theta}, \hat{\beta}, \hat{\alpha}, \hat{\rho}\} = H_q\hat{\theta} + \Psi^{\text{T}}(q)\Phi^{-1}\left(Y - H_{\mathcal{Q}}\hat{\theta}\right)$ and variance of $\Sigma(q) = \mathbb{E}\Big\{\left(\Gamma_{\text{dB}}(q) - \overline{\Gamma}_{\text{dB}}(q)\right)^2 \mid Y, \hat{\theta}, \hat{\beta}, \hat{\alpha}, \hat{\rho}\Big\} = \hat{\alpha} + \hat{\rho} - \Psi^{\text{T}}(q)\Phi^{-1}\Psi(q)$ can best characterize the received channel power (in the dB domain) when transmitting from an unvisited location $q \in \mathcal{W}$, where $H_q = [1\ -10\log_{10}(\|q - x_b\|)]$, $H_{\mathcal{Q}} = [\mathbf{1}_m\ -D_{\mathcal{Q}}]$, $\mathbf{1}_m$ represents the m-dimensional vector of all ones and x_b is the position of the remote station. Furthermore, $D_{\mathcal{Q}} = \left[10\log_{10}(\|q_1 - x_b\|)\ \cdots\ 10\log_{10}(\|q_m - x_b\|)\right]^{\text{T}}$, $\Phi = \Omega + \hat{\rho}\, I_m$ with Ω denoting a matrix with entries $\left[\Omega\right]_{i,j} = \hat{\alpha}e^{-\|q_i - q_j\|/\hat{\beta}}$ for $i, j \in \{1, \cdots, m\}$, $\Psi(q) = \left[\hat{\alpha}\, e^{-\|q - q_1\|/\hat{\beta}}\ \cdots\ \hat{\alpha}\, e^{-\|q - q_m\|/\hat{\beta}}\right]^{\text{T}}$, and ρ denotes the power of the multipath random variable (in dB). The symbol $\hat{}$ represents the estimation of the corresponding underlying parameters based on the a priori samples. See [9] for more details on the estimation of the underlying parameters and the performance of this framework with real data and in different environments.

[1]The channel here refers to the uplink from a position in the workspace to the remote station.

3. COOPERATIVE ROBOTIC BEAMFORMING WITH PERFECT CHANNEL KNOWLEDGE

Consider N unmanned vehicles in the workspace $\mathcal{W} \subseteq \mathbb{R}^2$. Let $d_i(x_i) = \|x_i - x_i^0\|_2$ be the distance traveled by robot i with x_i^0 and x_i denoting the initial and final position of robot i respectively. $\Gamma(x_i)$ is the channel power when robot i transmits from position x_i, and P_T represents the total joint transmit power constraint of the robots.[2] All the robots have a copy of the message that needs to be communicated. The robots then employ distributed transmit beamforming [8], which results in the total received power: $P_R = \sum_{i=1}^N p(x_i)$, where $p(x_i) = P_T\Gamma(x_i)$. We then have the following optimization for robotic path planning and cooperative communication:

$$\begin{aligned} &\text{minimize} && \kappa_M \sum_{i=1}^N d_i(x_i) \\ &\text{subject to} && \sum_{i=1}^N p(x_i) \geq P_{R,\text{th}}, \end{aligned} \tag{1}$$

where $x_i \in \mathcal{N}(x_i^0)$, $\mathcal{N}(x_i^0) \subseteq \mathcal{W}$ is the neighborhood around x_i^0 that the i^{th} robot is constrained to move in (if no constraints, the set will be the whole space) and $P_{R,\text{th}}$ is the minimum required received signal power at the remote station for a successful communication, imposed by the Bit Error Rate (BER) requirement. The optimization variables are x_is, the final positions of the robots. The optimization problem of (1) then aims at finding the positions that the robots should move to which minimizes the total motion energy and satisfies the required received power at the remote station.

3.1. Optimum Solution

In this section we show how the optimization problem of (1) can be posed as a multiple-choice knapsack problem, which can be solved optimally for several cases that arise in practice.

We discretize $\mathcal{W}$ into M cells with centers $r_j \in \mathcal{W}$, for $j \in \{1, \cdots, M\}$. Equation (1) can then be formulated as

$$\begin{aligned} &\text{minimize} && \kappa_M \sum_{i=1}^N \sum_{j \in \mathcal{N}_i} d_{ij}x_{ij} \\ &\text{subject to} && \sum_{i=1}^N \sum_{j \in \mathcal{N}_i} p_j x_{ij} \geq P_{R,\text{th}} \\ &&& \sum_{j \in \mathcal{N}_i} x_{ij} = 1,\ x_{ij} \in \{0,1\},\ \forall\, j \in \mathcal{N}_i,\ \forall\, i, \end{aligned} \tag{2}$$

where $d_{ij} = d_i(r_j)$ is the distance to cell j, $p_j = p(r_j) = P_T\Gamma(r_j)$, and $\mathcal{N}_i \subseteq \{1, \cdots, M\}$ is the set of cells present in $\mathcal{N}(x_i^0)$. We refer to (2) as the Motion Energy Minimization Problem (MEMP), with the optimal value of MEMP_{OPT}.

Lemma 1. *MEMP can be posed as a multiple-choice knapsack problem (MCKP).*

Proof. Let $\{\pi_{ij}\}$, $\{w_j\}$ be variables defined as $\pi_{ij} = \max_{k \in \mathcal{N}_i} d_{ik} - d_{ij}$ and $w_j = \max_k p_k - p_j$. Note that $\sum_{i=1}^N \sum_{j \in \mathcal{N}_i} \pi_{ij}x_{ij} = \sum_{i=1}^N \max_{k \in \mathcal{N}_i} d_{ik} - \sum_{i=1}^N \sum_{j \in \mathcal{N}_i} d_{ij}x_{ij}$ and $\sum_{i=1}^N \sum_{j \in \mathcal{N}_i} w_j x_{ij} = N\max_k p_k - \sum_{i=1}^N \sum_{j \in \mathcal{N}_i} p_j x_{ij}$. MEMP of (2) can then be posed as

[2]Note that the formulation and analysis can be extended to the case where individual transmit power constraints are enforced instead, which will be the subject of our future papers.

$$
\begin{aligned}
\text{maximize} \quad & \sum_{i=1}^{N} \sum_{j \in \mathcal{N}_i} \pi_{ij} x_{ij} \\
\text{subject to} \quad & \sum_{i=1}^{N} \sum_{j \in \mathcal{N}_i} w_j x_{ij} \le c \\
& \sum_{j \in \mathcal{N}_i} x_{ij} = 1,\ x_{ij} \in \{0,1\},\ \forall j \in \mathcal{N}_i,\ \forall i,
\end{aligned} \tag{3}
$$

where $c = N \max_k p(r_k) - P_{R,\text{th}}$ and the variables are x_{ij}. □

Equation (3) is the standard form of a well-studied optimization problem known as multiple-choice knapsack problem (MCKP) [10], [15]. Although MCKP is NP-hard, an exact solution can be efficiently achieved for several cases that arise in practice. In this paper, we thus utilize the minimal algorithm developed by Pisinger [10] to solve MCKP optimally.

Remark 1. *It is easy to show that the optimal values of the two formulations are related as* $\mathit{MEMP}_{\mathit{OPT}} = \kappa_M \left(\sum_{i=1}^{N} \max_{k \in \mathcal{N}_i} d_{ik} - \mathit{MCKP}_{\mathit{OPT}}\right)$.

4. COOPERATIVE ROBOTIC BEAMFORMING WITH PROBABILISTIC CHANNEL LEARNING

As was discussed earlier, the channel power over the workspace will not be known to the robots. Instead, the robots will utilize the probabilistic prediction approach of Section 2.2.2 to assess the channel at an unvisited location using a small number of a priori samples. The original robotic beamforming problem of (1) can be posed as follows in a stochastic setting:

$$
\begin{aligned}
\text{minimize} \quad & \kappa_M \sum_{i=1}^{N} d_i(x_i) \\
\text{subject to} \quad & \Pr\left(\sum_{i=1}^{N} p(x_i) < P_{R,\text{th}}\right) < \Pr_{\text{out}},
\end{aligned} \tag{4}
$$

where $x_i \in \mathcal{N}(x_i^0)$, $\Pr(.)$ denotes the probability of the argument, $p(x_i) = P_T \Gamma(x_i)$ with $\Gamma(x_i)$ being the random variable denoting the channel power when robot i transmits from x_i and $\Pr_{\text{out}}$ is the maximum tolerable outage probability.

As seen in Section 2.2.2, the received channel power (in dB) is best modeled as a Gaussian random variable i.e. $\Gamma_{\text{dB}}(x_i) \sim \mathcal{N}(\overline{\Gamma}_{\text{dB}}(x_i), \Sigma(x_i))$. $p(x_i) = P_T\Gamma(x_i)$ is then modeled as a lognormal random variable, i.e. $10 \log_{10} p(x_i) \sim \mathcal{N}(\mu(x_i), \sigma^2(x_i))$ where $\mu(x_i) = 10 \log_{10} P_T + \overline{\Gamma}_{\text{dB}}(x_i)$ and $\sigma^2(x_i) = \Sigma(x_i)$. The received signal power $P_R = \sum_{i=1}^{N} p(x_i)$ is thus the sum of lognormal random variables. Lognormal is a good approximation for the distribution of the sum of lognormal random variables [16], [17]. Then, we have $10 \log_{10} P_R \sim \mathcal{N}(\mu_{\text{sum}}, \sigma^2_{\text{sum}})$, where μ_{sum} and σ_{sum} can be found by employing the Fenton-Wilkinson method [17] wherein the first and second central moments of P_R and $\sum_{i=1}^{N} p(x_i)$ are equated. Then, the outage probability condition in (4) can be expressed as $\mu_{\text{sum}} + \sigma_{\text{sum}} Q^{-1}(1 - \Pr_{\text{out}}) \ge P_{R,\text{th,dB}}$, where $Q(.)$ denotes the Q function. The optimization problem (4) can then be solved by using existing optimization toolboxes.

Note that in general $p(x_i)$s may not be uncorrelated depending on how close any two robots get to each other. While we can express μ_{sum} and σ_{sum} as a function of individual mean and variance for the general case of correlated lognormal random variables, in order to reduce the computational complexity, we assume uncorrelated $p(x_i)$s in Section 5. However, each node still learns the underlying channel correlation parameters, as discussed earlier, and does not assume uncorrelated channels in the learning process.

4.1. Approximated Problem Via Knapsack Posing

The optimization problem (4) can be approximated with a deterministic equivalent by using the predicted mean and variance. More specifically, we can approximate $p(r_j)$ with $10^{(\mu(r_j) - \eta\sigma(r_j))/10}$, for some constant η in (2). This approximated problem can be solved using knapsack posing, as discussed in Section 3.1. We next find an η that guarantees a solution satisfying $\Pr_{\text{out}}$.

As in Section 3.1 we partition $\mathcal{W}$ into M segments with centers $r_j \in \mathcal{W}$. $10 \log_{10} p(r_j)$ is distributed as $\mathcal{N}(\mu(r_j), \sigma^2(r_j))$. Define $\tilde{p}_j = 10^{(\mu(r_j) - \eta\sigma(r_j))/10}$ for some constant η. Replace p_j by $\tilde{p}_j$ in (2). Let $j_i \in \mathcal{N}_i$ be the index of the cell chosen by agent i. The probability of successful transmission is then $\Pr\left(\sum_{i=1}^{N} p(r_{j_i}) \ge P_{R,\text{th}}\right) \ge \Pr\left(\sum_{i=1}^{N} p(r_{j_i}) \ge \sum_{i=1}^{N} \tilde{p}_{j_i}\right) \ge \Pr\left(p(r_{j_i}) - \tilde{p}_{j_i} \ge 0,\ \forall i\right) = \prod_{i=1}^{N} \Pr\left(p(r_{j_i}) \ge \tilde{p}_{j_i}\right) = [Q(-\eta)]^N$ since $\sum_{i=1}^{N} \tilde{p}_{j_i} \ge P_{R,\text{th}}$ and $p(r_{j_i})$s are assumed to be uncorrelated. Imposing a probability of outage of $\Pr_{\text{out}}$ gives us $\eta = -Q^{-1}\left((1 - \Pr_{\text{out}})^{1/N}\right)$ while the resulting MCKP problem is efficiently solved. However, we may not be able to find a feasible solution since this approach is overly conservative.

5. SIMULATION RESULTS

We generate a realistic 2D channel over a rectangular workspace, using the method described in [14]. The underlying parameters are chosen as follows $P_{T,\text{dB}} = 0$ dBm, $K_{\text{dB}} = -40$ dB, $n_{\text{PL}} = 3$, $\alpha = 5$, $\beta = 3$ m and $\rho = 1.3$. The minimum required received power is taken as $P_{R,\text{th,dB}} = -70$ dBm.

We first consider the case where the channel is perfectly known to the robots (Section 3). We implement D. Pisinger's minimal algorithm [10] to solve the resulting MCKP and obtain the optimal solution. Fig. 1 shows the solution for a sample channel realization. It can be seen that two of the nodes incurred non-negligible motion energy to find spots better for cooperative beamforming. Next, we discuss the underlying trends as a function of the channel parameters by running several simulations for each set of parameters. Fig. 3 shows the total distance traveled by all the nodes as a function of the path loss exponent. For each exponent, several channels are generated with the corresponding exponent and the rest of the parameters as mentioned before. The performance is then averaged over these runs as well as over the initial distribution of the positions over the space. We can see that as the path loss exponent increases, the total traveled distance increases due to lower channel power values. Fig. 4 shows the trend as the shadowing power increases. It can be seen that the total distance traveled decreases due to the fact that as the shadowing power increases, spatial variation increases. Thus, a node can find a good spot with lesser movement. Fig. 5 shows the trend as a function of the shadowing decorrelation distance. As the decorrelation distance increases, channel gets uncorrelated slower. It can be seen that the nodes have to travel less as the decorrelation distance gets smaller similar to the impact of shadowing power increase. Note that the remote station is out of the workspace on the lower left corner.

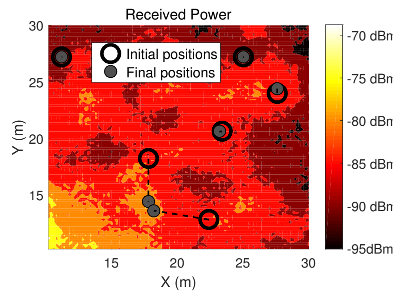

Fig. 1. Distributed Robotic Beamforming – Case of perfect channel knowledge. See the pdf for a color illustration.

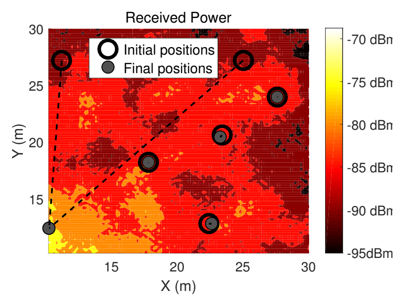

Fig. 2. Distributed Robotic Beamforming – Case of probabilistic channel learning.

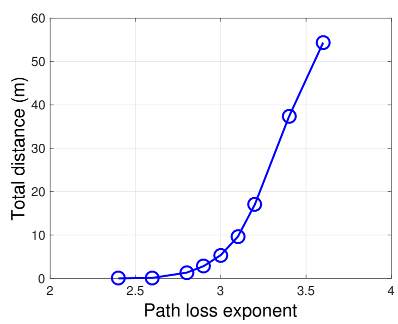

Fig. 3. Average total distance traveled as a function of path loss exponent (n_{PL}) for the known channel case.

Next, we simulate the case where the robots probabilistically predict the channel (optimization problem (4)). The channel is generated in a similar manner described above and the outage probability is taken as $\text{Pr}_{\text{out}} = 0.1$. The channel is then predicted based on 5% randomly-spaced prior measurements over the workspace. This channel predictor as well as the Fenton-Wilkinson method are used in conjunction with the *fmincon* function of MATLAB to obtain a solution.[3] Fig. 2 depicts the solution for the sample channel realization and

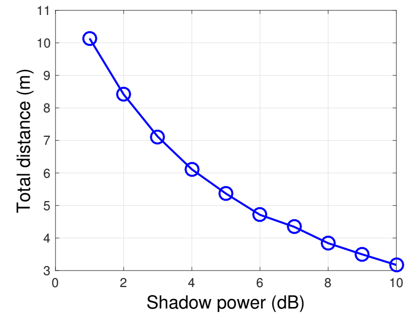

Fig. 4. Average total distance traveled as a function of the shadowing power (α) for the known channel case.

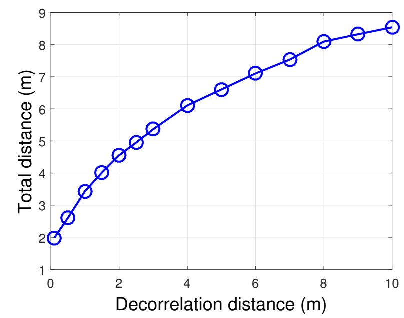

Fig. 5. Average total distance traveled as a function of the decorrelation distance (β) for the known channel case.

robot initial positions of Fig. 1. The robots move more in this case as their assessment of the channel is not perfect.

Due to the uncertainty of channel prediction, there could be cases where the final positions do not satisfy the received required power of (4), i.e. $P_T \sum_{i=1}^{N} \Gamma(x_i) < P_{R,\text{th}}$. Then, the nodes have to increase their total joint transmit power budget beyond the nominal value of P_T to $\frac{P_{R,\text{th}}}{\sum_{i=1}^{N} \Gamma(x_i)}$. The more conservatively Pr_{out} is selected, the lower the chance of a solution that does not satisfy the received power requirement. However, this comes with an increase in the chance of not finding feasible solutions. Overall, several factors such as channel prediction and learning affect the final solution and performance. Further analysis of them as well as analyzing the trends of the solution is an avenue for future work.

6. CONCLUSIONS

We considered the scenario where a team of mobile robots are tasked with transmitting a message to a remote station using distributed beamforming, while minimizing the total motion energy and satisfying a given reception quality constraint. When the channel is perfectly known, we showed how to pose this as a multiple-choice knapsack problem. We then extended our approach to the case of unknown channel over the space. More specifically, we showed how probabilistic channel assessment can be integrated with path planning for distributed robotic beamforming under motion energy constraints. Finally, we presented extensive simulation results in realistic communication environments.

[3] Note that (4) is a nonconvex optimization problem.

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