Discrete - Time Signals and Systems

Introduction

Yogananda Isukapalli
Function of an independent variable, in mathematical Sense

Signal: **Something** that carries information

A Pattern of variations of a physical quantity
That can be manipulated, stored or transmitted
By physical processes
Some Examples

• Speech Signals
• Audio Signals
• Video or Image Signals
• Biomedical Signals
• Radar Signals
• Seismic Signals
• Sonar Signals, etc…

Transducers help in giving Signals many Convenient representations
Mathematical Representation of Signals

Function of an independent variable, in Mathematical Sense

- **Independent variable can be**
  - ‘time’, ex. Speech signal
  - ‘spatial co-ordinates’, ex. Image
  - ‘time and space’, ex. Video
  - ‘numerical index’

- **Function can be,**
  - Of many Dimensions
  - Continuous, Notation “( )”, as in x(t)
  - Discrete, Notation “[ ]”, as in x[n]
‘Speech Signal’ shown in fig.1.1 is an example of a one-dimensional continuous-time signal.

Fig.1.1
Sampled version of the shaded region is shown in Fig1.2
‘Speech Signal’ shown in fig.1.2 is an example of a one-dimensional continuous-time signal.

\[ s[n] = s(nT_s) \]

\( T_s \) is the sampling period; \( T_s = 1/8 \text{ msec} \)
‘Image Signal’ shown in fig.1.3 is an example of a ‘two-dimensional discrete-spatial’ signal. The gray-scale image is represented by a two-dimensional sequence of an array of numbers and would be denoted as $P[m, n]=p[m\Delta_x, n\Delta_y]$, where ‘m’ and ‘n’ would take integer values only, $\Delta_x$ and $\Delta_y$ are the sampling rates along horizontal and vertical directions.
Some Real World Signals

Sun Spot Data Series

Number of Sun Spots

Years

1700 1750 1800 1850 1900 1950

Sun Spot – Frequency Spectrum

Amplitude

Frequency

-1 -0.5 0 0.5 1

Canadian Lynx Data Series

Number of Canadian Lynx

Years

1820 1840 1860 1880 1900 1920 1940

Canadian Lynx – Frequency Spectrum

Amplitude

Frequency

-1 -0.5 0 0.5 1
Real World Signals Contd ....

Fig. 1.5

Laser Intensity Data

ECG Data

x $10^4$ Laser Data – Frequency Spectrum

x $10^4$ ECG Data – Frequency Spectrum
Some Analytical Signals

Fig. 1.6

- **Impulse**
  - Time domain: Constant amplitude with a single impulse.
  - Frequency domain: Constant amplitude with a peak at the impulse location.

- **Step Function**
  - Time domain: Constant level followed by a step change.
  - Frequency domain: Concentrated energy at the step frequency.

**FFT of Impulse – Magnitude**
- Constant amplitude over frequency range.

**FFT of Step – Magnitude**
- Energy concentrated at specific frequencies related to the step, with higher amplitudes.
Analytical Signals Contd …

Fig. 1.7

**Pulse**

- Amplitude
- Time

**FFT of Pulse – Magnitude**

- Amplitude
- Frequency

**Sine**

- Amplitude
- Time

**FFT of Sine – Magnitude**

- Amplitude
- Frequency
• Many real-world signals are analog in nature
• Discrete-time signals have many desirable properties
• A continuous signal $s(t)$ sampled at $T_s$ becomes a discrete-time signal, mathematically represented as $s[t]$
• Discrete-time signals having been quantized and converted into sequences of digital numbers become digital signals
An Illustration of different forms of signals

Analog, sampled, quantized and digital signals.

Continuous-time signal

Discrete-time signal

Quantized signal 8 levels

Digital signal 8 levels
System

Something that can manipulate, change, record
Or transmit signals

In general, systems operate on signals to produce
new signals or new signal representations

Mathematically \[ y(t) = \tau\{x(t)\} \]
Continuous-time system

Both the input and output are continuous-time signals

\[ x(t) \xrightarrow{\tau\{x(t)\}} y(t) = x(t)^2 \]

Output is the squared value of input

Fig.1.9
Discrete-time system

Both the input and output are discrete-time signals

\[ x[n] \xrightarrow{\tau\{x[n]\}} y[n] = |x[n]| \]

Output is the absolute value of input

Fig.1.10
“Sampler” is a system which converts a continuous–time signal into a discrete one.

An example of a sampled signal (ideal sampling). The values of the signal samples are equal to those of the original analog signal at the sampling instants.

**Fig. 1.11**
The above figure shows a complete processing of analog signal, the analog signal is converted into a digital one, processed and converted back to an analog signal.
Example

Application of DSP to recover damaged sound tracks: Sampled Voice of *Darth Vader* from ‘*Empire Strikes Back*’ Saying “Don’t Fail me again”.

Noisy Sound

Fig.1.13
Example Contd...

Typically Sounds recorded prior to 1980 were all in analog format, because of which, due to age and wear of the recorded medium, there is a distinguishable loss in quality of sound. Now in-order to recover back as much as the original sound as possible we convert the noisy sound into a digital format and apply DSP techniques for removal of noise. After Removal of Noise ;

Sound after removal of Noise

Here we could clearly notice the effect of filtering

Fig.1.14
Another Sound clip from the movie *Return of the Jedi*:

Here we can clearly hear the noise which has crept in to the original soundtrack. Now we process this signal using DSP techniques to get rid of the noise.

We can clearly notice a marked change in quality of the sound after processing the signal.
How are we Achieving this?

Most of the energy in Speech Signals lies in the frequency band of 0 - 3000 Hz. Utilizing this fact we design a digital filter to remove of all the high frequency components in the signal of our interest, thus eliminating the unwanted noise.

Filter Specification

<table>
<thead>
<tr>
<th>Equiripple</th>
<th>Lowpass</th>
</tr>
</thead>
<tbody>
<tr>
<td>f1: 0.25</td>
<td>R: 2.165</td>
</tr>
<tr>
<td>f2: 0.285</td>
<td>R: 40</td>
</tr>
<tr>
<td>Fs: 1</td>
<td>Order: 34</td>
</tr>
</tbody>
</table>

Fig.1.15
An Example of DSP in Image Processing

Use of DSP techniques for Zooming instead of using expensive & bulky Zoom Lens

Camcorders use DSP Techniques to achieve a greater zoom than that available on the camera lens. Although this technique deteriorates the image quality at very high zoom factors, but when compared to the cost & weight of lens required for that zoom factor, the cost & weight of the DSP chip used for this application is very marginal.
**Image Processing Example**

Zooming onto the area shown

In this technique the existing data is interpolated to get the new zoomed in version of the image. This example would compare with the process of up-sampling of 1-dimensional signals.
How is that done?

Every Image is represented by points known as pixels, each pixel has three values ranging from 0 - 1 for each of the three primary colors “RGB”, for a color image and only by a single value for a Black & White image. Now when an image is zoomed in by a certain factor no new information is added in rather the existing information of the pixels present in the area of interest are replicated by the zoom factor to give us the zoomed version of the image.
Applications of Signal Processing

- Communication Systems
- Speech & Music Processing
- Image Processing
- Medical Imaging
- Biomedical Signal Processing
- High speed Modems
- Closed Loop Control systems
- Radar/Sonar signal analysis
- Real Time Measurement & Instrumentation
Applications contd..

- **SPACE**
  - Space photograph enhancement
  - Data compression
  - Intelligent sensory analysis by remote space probes

- **MEDICAL**
  - Diagnostic Imaging (CT, MRI, ultrasound)
  - Electrocardiogram analysis
  - Medical image storage/retrieval

- **COMMERCIAL**
  - Image and sound compression
  - Movie special effects
  - Video conference calling

- **TELEPHONE**
  - Voice and data compression
  - Echo reduction
  - Signal multiplexing
  - Filtering

- **MILITARY**
  - Radar
  - Sonar
  - Ordinance guidance
  - Secure communication

- **INDUSTRIAL**
  - Oil and mineral prospecting
  - Process monitoring & control
  - Nondestructive testing
  - CAD and designing tools

- **SCIENTIFIC**
  - Earthquake recording and analysis
  - Data acquisition
  - Spectral analysis
  - Simulation and modeling
Given the advent of DSP chips, DSP technology can now be found in such devices as mobile phones, multimedia computers, video recorders, CD players, hard disc drive controllers and modems. It will soon even replace analog circuitry in TV sets and telephones. Most of these practical applications exploit two key attributes of DSP technology: signal compression/ decompression and real-time operation.

Signal compression and decompression is used in a variety of applications. In CD systems, for example, the music recorded on the CD is in a compressed form (to increase storage capacity). It must be decompressed in order for the recorded signal to be reproduced.
DSP technology enables the signal to be compressed and decompressed resulting in a cleaner, crisper signal.

- Signal compression is also used in digital cellular phones to allow a greater number of calls to be handled simultaneously within each local "cell". This compression technology allows people not only to talk to one another by telephone but also to see one another on the screens of their PCs, using small video cameras mounted on the computer monitors, with only a conventional telephone line linking them together.
• The architecture of a DSP chip is designed to carry out complex mathematical operations incredibly fast, processing up to tens of millions of samples per second, to provide real-time performance. The real-time performance results from the ability to process a signal "live" as it is sampled and then output the processed signal, for example to a loudspeaker or video display. Most of the practical examples of DSP applications, such as hard disc drives and mobile phones, demand real-time operation.
Advantages of DSP

- **Guaranteed Accuracy** (determined by the number of bits)
- **Superior Performance** (Than analog signal processing)
- **Perfect Reproducibility** (no variations due to component tolerances)
- **No Drift in performance with temperature & age**
- **Greater Flexibility** (wider applications with minimal changes in hardware)
- **Immunity from Noise**
Reference